1. INTRODUCTION

Increasing consumer demands for door-to-door service (door-to-door cargo delivery) makes the relevant requirements to the participants of the transportation process like a freight-forwarder companies which serve and extend the services what provided by an air company for the benefit of commercial customers. Freight-forwarding companies are freight transportation agents, perform services for customs clearance, handling of goods, mutual settlements, etc. A huge number of freight-forwarder companies which serve air-freight operations, offer road and railway transportation. Large freight-forwarder companies have own terminal networks what consist of cargo distribution and sorting centers, organize distribution centers at the airports and railway stations, build their own network of road, air, sea and railway transportation. Logistics and transportation companies operate as a multimodal transport operator (MTO) by giving the client their own (not an agency) waybill.

Among the variety of services offered today by multimodal operators, there are two main types of service: express delivery and economy delivery. In the first case, the basic requirement of customers is the minimum time and reliability of delivery with reasonable rates. In the second case - a minimum transportation cost at reasonable requirements to transit time. Express delivery uses air transport mostly. Road transport is used for initial and final stage of express door-to-door transportation. Road and rail transport is used less often at initial and intermediate stages (between two air transportation) express delivery, only in certain areas and coordinated schedule. Conversely, economy shipping uses mainly road, sea, inland waterway and railway transport for transportation between terminals. Air transport is used only in certain areas.

Competitiveness of multimodal operators is determined by their possibilities to present the widest range of services for today and to comply with the diverse requirements of customers. Economy shipping is relatively cheap but long and express delivery is fast, but expensive. They can be considered as two extreme options offered by MTO services. There are many intermediate cost and time of delivery options between them. These options are described here by the concept of the family of L-shortest routes on the network MTO for different values of parameters \( L = 1, 2, \ldots \) and a variety of criteria as a the minimum cost, minimum distance or the time of delivery. Optimal routes match value \( L = 1 \). If \( L = 2 \) route is computed by the second most important (by rank) the objective function and etc.

In this article we develop a mathematical models of the multimodal transport terminals networks (TTN), which consist of four modes of transport (road, air, sea and rail). This models and the algorithms help to compute the optimal L-shortest routes, what are optimal according to various criteria including customer preferences. Also they help to simulate the processes of delivery and distribution of freight flows to the optimal routes with a specified schedule. The TTN modeling with different modes can provide data describing the effectiveness of a decision management solutions (such as flight schedules), related to the organization of transportation by a carrier company, reveal the most "narrow" places of the organization of freight forwarding services in transport terminals network.

The issues of modeling and optimization what are close to the purposes of this article are considered in [1-6] articles.
2. **MATHEMATICAL MODEL OF THE MULTIMODAL TRANSPORT TERMINALS NETWORK. BASIC VERSION**

*The structure of the network.* Transport terminals network is a set of nodes and arcs. Network nodes are its terminals. Network arcs are transport routes which connecting the two terminals. Nodes are the distribution centers (DC), which handle and sort cargo for transportation. There are two types of DC – head (hub) distribution center and gateway DC (GDC). Head DCs work with customers directly (shipper and consignee). Also they provide auto-service by regular road routes between terminals, sort, consolidate and collects the goods from the shipper (the initial stage of carriage) for the final recipients delivery [2]. The gateway distribution centers are designed to transfer (and receive) cargo from DC to air, sea, and rail carrier. They are located at airports, seaports and train station. Air, sea and rail routes link only respective type of gateway DC between each other. Transportation between head DC and GDC are carried out by road.

Basic transport terminals network, what is considered in the examples of this article and implemented in the software package, consists of three interrelated parts - the central, eastern and western.

The central part of the transport terminals network covers the European part of Russia and consists of 17 head DC what are linking their regular networks of road routes, 8 GDC to air network and air routes which linking the gateway terminals of air network. Terminals are located in St. Petersburg, Moscow (North and South terminals), Voronezh, Rostov-on-Don, Krasnodar, Nizhny Novgorod, Kazan, Yaroslavl, Samara and Yekaterinburg.

The eastern part of transport terminals network consists of 7 head DC in major cities of Siberia and the Far East of Russia, 7 gateway DC to aviation network (at local airports), 7 gateway terminals to the rail network (at the local railway station) and connecting them network of routes. The eastern part connects to the central part of network through the head DC in Yekaterinburg and Omsk, and then in the direction of the Trans-Siberian Railway to the head DC in Omsk to head DC in Novosibirsk, Krasnoyarsk, etc. up to Vladivostok.

The western part of the transport terminals network consists of 7 head DC in Western Europe (Helsinki, Berlin, Frankfurt am Main, Hamburg, Rotterdam, Paris, London), 6 gateway DC to aviation network, 1 rail gateway terminal (at the Berlin train station), 6 sea gateway terminals and connecting them the network of routes (fig. 1).

![Western part TTS.](image-url)
In total, transport terminals network basic version contains \( n = 31 \) head DC, \( \text{nav} = 21 \) terminals of approach to aviation network, 6 sea GDC and \( n\text{RS} = 11 \) terminals of approach to the rail network.

**Transport terminals network schedule and rates.** Road, air and rail transport routes are described equally below. We can add array \( \text{ScheDat}[i, j] \) in the logistics information system «MultiTransGlobal», what contains information about the routes interterminal transportation schedule (ScheduleData). Here “\( i \)” is routing code in the schedule and “\( j \)” is parameter routing. Variables like \( \text{ScheDat}[i, 1] \) are denote the code of departure DC, \( \text{ScheDat}[i, 2] \) - destination DC code, \( \text{ScheDat}[i, 3] \) – code of the type of scheduling, \( \text{ScheDat}[i, 4] \) - the time of departure, \( \text{ScheDat}[i, 5] \) – transit time of route. All timing parameters are set in minutes. Schedule is attached to the weekly cycle. Schedule type means interterminal routes with different departure days of the week, for example- code 0 means the daily schedules of departure, code 1 - 6 days of departure (except Sunday) etc/

Fares (USD / kg) are given by the matrix of cost:

\[
C = C[i, j]
\]

(1)
diagonal elements of \( C[i, i] \), which means the rates for terminal handling in the \( i \)-th terminal, and the off-diagonal elements when \( i \neq j \) - interterminal rates for transportation.

A matrix of distances on roads (km) between nodes (head DC and DC of approach) of the network is also refers to the network. If the terminals are not connected by direct routes, the distance and the corresponding elements of cost matrix are equal to infinity.

### 3. L-OPTIMAL TARIFF AND DISTANCE ROUTING

We introduce the quantities \( U_r^k[i] \) as a length of \( r \)-th shortest route, \( r = 1,2, \ldots, L \) from the \( i \)-th node to the fixed end node \( i_k \) of the network with numbers of intermediate nodes up to \( k \). Generic term "node", "length", "shortest route" are used in the general definitions here and throughout the article. But the route length means its value (price, $ / kg) or distance (rally, km) in this section, and length means the time of delivery in the next section. When the route computation parameter \( k = 0,1, \ldots, (n-2) \), it also has the meaning of the iteration number, where \( n \) is the total number of nodes. Values \( U_r^k[i] \) are defined by the following recursive equations:

\[
U_r^k[i] = \min_r \{C[i, i] + C[i, j] + U_r^{(k-1)}[j]\}, i = 1,2, \ldots, n
\]

(2)

Here and below, the symbol \( \min_r \) is the \( r \)-th smallest \((r \)-th minimum grade\) set of numbers in curly brackets. The set of numbers is formed by sorting of all nodes \( j \neq i \), connected by arcs (interterminal routes) with the \( i \)-th node and the length of \( \mu \)-s shortest routes \( U_r^{(k-1)}[j] \) from node \( j \) to node \( i_k \), what is computed on the previous \((k-1)\)-th iteration. The initial conditions for these equations are determined with \( k = 0 \) by the expression

\[
U_r^0[i] = C[i, i_k] + C[i, i] + C[i_k, i_k], r = 1
\]

(3)

When \( r \neq 1 \), or if the nodes \( i \) and \( i_k \) are not directly related arcs, the \( U_r^0[i] \) meanings are set equal to the infinity.

Computation of the L-optimal routes leading from a given initial node \( i_0 \) to a fixed end node \( i_k \) is performed after the computation of the target functions \( U_r^k[i] \) with \( k = (n-2) \). The computations are similar to the procedure described in [2]. Bust rank values \( r = 1, 2, \ldots, L \) defines a family of L-optimal tariff routes.

L-optimal by a minimum distance routes are computed by the algorithms, putting the cost of the matrix equal to the distance of distances by road between terminals. L-optimal routes by tariffs and distance (as opposed to the optimal time) do not depend on ScheduleData of the network, the time and day of cargo arrival at the departure terminal.
4. COMPUTATION OF L-OPTIMAL TIME OF DELIVERY ROUTES

This type of routes tied to a schedule of regular interterminal transportations what depend on parameters of the schedule and the arrival of the cargo to the original departure terminal. Algorithm of the search is complicated by the emergence of new variables. Consider the quantities $U_r[l,t,i]$ that is the length of $r$-th shortest route, $r = 1, 2, ..., L$ (a minimum time of delivery) from the $i$-th terminal to the end terminal $i_k$, providing that the route contains no more $k$ intermediate terminals, and cargo arrived to the $i$-th departure terminal at the time $t$ days $l$-th day of the week, $l = 1, 2, ... 7$.

$T$ is discrete variable, what appeared as a result of the discretization of continuous time of the day with a step $dt$. There is $dt = 10 \text{ min}$, discrete time is set to $0, 1, ..., 144$ in the program. Further, all the temporal characteristics of the transport terminals network (time of arrival at the terminal, departure, duration of the interterminal routes, and the meaning of function $U_r[l,t,i]$) is measured in minutes. Variables $U_r[l,t,i]$ are related by the equations

$$
U_r[l,t,i] = \min_r \{T[l,t,i] + U_{r-1}[l,t,j]\}.
$$

Here $T[l,t,i,j]$ is the time of cargo delivery from the $i$-th to the $j$-th terminal, by a schedule what is determined by the regular service, provided that the cargo have arrived to the $i$-th terminal with arrival parameters $(l, t)$. If there is no direct flight between the $i$-th and $j$-th nodes in the, this value is equal to infinity; $l_j$ and $t_j$ are day and the daily arrival time to the $j$-th terminal. The parameter $k = 1, ..., (n-2)$ has the same meaning as in the equation (1).

The initial conditions with $k = 0$ for equations (3) have the form

$$
U_0[l,t,i] = T[l,t,i], \text{ if } r = 1.
$$

If $r > 1$, or there is no direct interterminal flights between the $i$-th and $j$-th points, the values of the functions $U_r[l,t,i]$ are set equal to infinity.

L-optimal routes laying by delivery time is the reverse motion procedure [2]. Time optimal routes depend on the day of the week and the time of cargo arrival to the departure terminal in contrast to the optimal tariff routes. In addition, the existence and uniqueness of the routes can be violated.

5. COMPUTATION OF FREIGHT FLOWS IN THE THREE-MODAL TRANSPORT TERMINALS NETWORK

Input freight flows are modeled in the matrix form

$$
X_{inp} = X_{inp}[i,j], \text{ } i, j = 1, 2, ..., n
$$

elements $X_{inp}[i,j]$, which determine the amount of weight (kg / day) sent from the region maintained by the $i$-th distribution center (DC) to the $j$-th terminal region. The input traffic is computed by the equation

$$
X_{inp}[i,j] = \alpha \text{Population}[i] \lambda_{i,j},
$$

where the parameter $\alpha$ is the mean amount of weight (kg / (1000 people a day)) sent by a company a day from every 1,000 people in the region, the array $\text{Population}[i]$ contains population data of the region $i$-th DC in thousands of units, coefficient $\lambda_{i,j} \in (0,1)$ means proportion of the total freight flow what is coming from the $i$-th region and being routed to the $j$-th region. The example assumed $\alpha = 3 \text{ kg} / (1000 \text{ people a day})$. Coefficients $\lambda_{i,j}$ satisfies the normalization condition

$$
\sum_{j=1}^{n} \lambda_{ij} = 1, \lambda_{ij} = 0 \text{ for } i = j.
$$

Interterminal network freight flow (kg / day) is given by the matrix:

$$
NetFl = (NetFl[i,j]),
$$
its elements are computed as the total flow path in the direction of the $i$-th terminal to the $j$-th (the arc $(i, j)$) along the optimal route $M_{opt} [l, m]$, which connects the initial $l$-th and $m$-th end nodes

$$NetFl[i, j] = \sum_{l=1}^{n} \sum_{m=1}^{n} X_{inp}[l, m], l \neq m.$$ (8)

The summation of input flow $X_{inp}[l, m]$ is performed by all optimal routes $M_{opt} [l, m]$, that includes the arc $(i, j)$. Computing algorithm of the value of network flows performed by computation the (first) best route $M_{opt} [l, m]$ from node $l$ to node $m$, assigning to all the flow values $NetFl [i, j]$ of all the arcs along the route $M_{opt} [l, m]$ of the input flow $X_{inp}[l, m]$ and sum over all the values of the input $l$ and output $m$. The flow $NetFl [i, j]$ is the volume of traffic (traffic in kg / day), operated by the company in the direction of $(i, j)$. Its value is required for selecting the vehicle and flights planning, which ship in the direction from the $i$-th terminal to the $j$-th terminal.

Terminal (node) flows determine the operation of terminal network as transshipment points. Flows (total in both outgoing and incoming flights) of local shippers (index "ls") and consignees (index "lc") in the $i$-th node are determined by

$$X_{ls}[i] = \sum_{j=1}^{n} X_{inp}[i, j]$$ (9)

$$X_{lc}[j] = \sum_{i=1}^{n} X_{inp}[i, j]$$ (10)

6. COMPUTATION OF IN TERMINAL FREIGHT FLOW OF TRANSPORT TERMINALS NETWORK

The structure of the technological areas of sorting terminals includes $K$ receptions areas to serve the arriving flights, and the same number of shipping platforms maintaining departing flights. Receiving and shipping areas form unloading and loading fronts. A detailed description of the freight flow inside the terminals (head DC and gateway DC) is determined by the matrix

$$CrFl[ i ] = ( q_{rs} ), r,s=0,1,...,K,$$

where $i$ is the number (code) of the terminal, which is not considered in the notation of elements of the matrix, zero index is assigned to the local freight flows; $q_{0s}$ is a flow of local shippers which sent for loading to the flight of $s$ - th shipping area; $q_{r0}$ is a flow for local consignees arriving on the $r$-th arrival reception area; $q_{rs}$ - transit freight flow for $r, s = 1, ... K$, passing through the terminal from the flight what arrived to the $r$-th reception area to the flights what is departing from $s$-th shipment site. All flows have the dimension kg / day.

Flights from certain "adjacent" terminal (or a particular terminal) are assigned for each of the receiving and shipping areas. Denote $j_{opr} [r]$ as a code of adjacent terminal. The flight from this terminal is unloaded at $r$-th receiving area. $j_{opr} [s]$ is a code of adjacent terminal. The flight to this terminal is sent from $s$ -th shipping area. These flights form interterminal transport routes in directions $j_{opr} [r] \rightarrow i, i \rightarrow j_{opr} [s]$. With their help we can compute the flow.

Equals (6) - (8) are used in the computation, but the terms of summing are different every time. Flows $q_{0s}$ of local shippers are computed by formula (7); values of the outgoing flow from the $i$-th terminal which is sent to the $j$-th end terminal for the best route $M_{opt} [i, j]$, containing an initial route $i \rightarrow j_{opr} [s]$ are accounted. Flows for the local consignees $q_{r0}$ are distributed to the receiving area in accordance with the formula (8) similar, where the summation is only over the terminal $j$, coming from the best route which $M_{opt} [j, i]$ ends by route $j_{opr} [r] \rightarrow i$.

Transit (cross) $q_{rs}$ flows are computed by formula (6), where the sum of input flows sent from $l$-th terminal to the $m$-th is performed only if the optimal route what is connecting them $M_{opt} [l, m]$ contains two adjacent interterminal route $j_{opr} [r] \rightarrow i, i \rightarrow j_{opr} [s]$. 
7. LOGISTICS INFORMATION SYSTEM «MULTITRANSGLOBAL»

Previous algorithms are the basis for information and logistics system (ILS) «MultiTransGlobal», developed at the department of intermodal transport and logistics in Saint-Petersburg State University of Civil Aviation. Software package allows computing and displaying the entire set of L-optimal routes and their characteristics with a given direction (from the \(i\)-th to the \(j\)-th terminal): through rate of the route, delivery time and full-time schedule of the customer order transportation with the arrival-departure schedule at intermediate terminals. The client chooses the route that suits for him from a variety of L-optimal routes (fig. 2-4). The routes that are selected by the customer are the source for solving the problem of simulation transport terminals network. Network parameters are entered in the Main Menu mode "Basic version of the transport terminals network", "The schedule", "Online database".

![Fig. 2. Entering search parameters of L-optimal routes client items for multimodal transport.](image)

![Fig. 3. Multimodal route Saratov- Gavr.](image)
Route computation is carried out in the procedures and MatrShortRoute and OptTimeRouteNew. Imitation modeling block reproduces the minute change in the current data of the freight flow in the transport terminals network with time $dt$, the change of day and days of the week, the emergence of transportation orders on all terminal centers (31 orders per day at each of the n terminals are simulated in this example). Procedures which are forming unit perform the following functions:

- Modeling and storing of basic order parameters (DC of departure, destination, departure weight - procedure ShipmentSimulator), modeling of flight departure for the entire network in accordance with the schedule (procedure DepartureOrder);
- The distribution of orders for flights according to the selected transportation routes and forming cargo manifest for each flight, arrival flight schedule to the appropriate destination terminal (procedure ArrivalOrder);
- The order tracking on the terminal network with the issuance of notice of arrival to the intermediate terminal and a notice of delivery to the final destination terminal (procedure ArrivalProcessing);
- Computation of input (procedure InputFl), network (procedure NetworkFl) and node (procedure NodeFlow) freight flows.

8. CONCLUSION

The problem what is considered in this article covers the important modeling tasks, optimization of transport routes and freight flow computation in freight terminal networks of multimodal transport operators. The use of L-optimal routes with different ranks and different criteria (minimum rates and delivery time) provides effective formal selecting routes mechanism, allowing to realize the full transport potential of intermodal transport terminal network. Manager who is working with the customer with the L-optimal routes can offer the customer a range of a routes to suit the maximum extent hardly compatible requirements for cost and delivery time. Freight flow computations for estimated values of the input flows, which express the expected traffic demands, can properly allocate handled cargo resources of MTO - vehicles on interterminal routes, drivers and loaders inside the terminals. All the proposed algorithms are implemented in the form of appropriate software and have been effective.
Abstract

There are presented the mathematical models, routing algorithms and computation algorithms for cargo flows in transport-terminal road-air-sea-railways networks of multimodal transport operators.

Keywords: Multimodal cargo operators, transport-terminal networks, optimal delivery routing, terminal, interterminal and within terminal cargo flows, planning, algorithms, software.

Modelowanie matematyczne, optymalizacja i estymacja parametrów w sieci terminali transportu multimodalnego

Streszczenie

Zaprezentowano modele matematyczne, algorytmy routingu i algorytmy obliczania przepływów towarowych w sieciach terminali transportu drogowo-powietrznokolejowego operatorów transportu multimodalnego.

Słowa kluczowe: Przewozy ładunków, sieci terminali transportu multimodalnego, optymalne dostawy.

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