1. INTRODUCTION

It is well-known that to reach the stable competitive position in market an enterprise must promptly and flexibly react on demand fluctuations and the competitors’ behavior with the help of supply chain (SC) management conception. The main goal of SC functioning is material flows movement along whole supply chain from one supply chain to another one or to finite consumer [1, 2]. As it was noted in the book [2], “Logistics is essentially a planning orientation and framework that seeks to create a single plan for the flow of product and information through a business. Supply chain management builds upon this framework and seeks to achieve linkage and co-ordination between the processes of other entities in the pipeline, i.e. suppliers and customers, and the organization itself”. In dependence of number of suppliers involved in production process, types of finished product the all SC may be conditionally divided into three groups, denoted by the letters A, V, and T [3]. So-called VAT-classification of SC is based on how the material flow comes through the nodes of SC. The specific feature of the V type of SC is existence of small number of suppliers of the same type of complete set or materials which in result of production process are transformed into big number of types of finished product. The SC of the A type have the opposite structure. In this case the relatively small number of finished products may be manufactured from big number of kinds of complete set and raw material kinds. At last, the T type of SC is featured by small number of suppliers and division of whole production process into two stages: a) manufacturing and storage of main complete set with use of simple operations and b) the unity of same complete set in different combinations.

The study of special literature devoted to problem of modeling and optimization of such complicated (large-scale) systems like SC shows that there exist no much works on this topic. For example, in the books [4, 5] and in the works [6, 7] only simplest linear (with consequently connected nodes) configurations of SC were studied with the combination of discrete, dynamic or linear programming models and inventory control models of the Wagner-Whitin type.

2. RESEARCH PROBLEM AND RESEARCH METHODOLOGY

The aim of our paper is development and analysis of dynamic optimization model of the A type of SC. It may be considered as further development of the work [7] for the case of more complicated structure of SC. Our model includes a finite set of enterprises-suppliers, enterprise-manufacturer which produces several types of finished product, transportation network, and set of points of destination. In respect of demand at points of destination for the planning horizon we make two suppositions: a) demand is given and fixed, and b) demand is described by mutually independent random variables with the known probability densities.
Like in the work [7] we will use the generalization of the Wagner-Whitin classical inventory control model [4, 5]. Below, to avoid too bulky analytical constructions we will restrict ourselves by consideration of more simple particular case of the A type of SC including three levels only. The static variant of such type of SC model, in the term of multi-index linear programming problem, was analyzed in the article [8].

3. OPTIMIZATION MODEL FOR FIXED DEMAND OVER PLANNING HORIZON

Let us consider the S enterprises-suppliers manufacturing their own complete set for further assembling the finished products by the unique enterprise-manufacturer. The s-th enterprise-supplier manufactures the L_s kinds of complete set from the R_s kinds of raw materials and other industrial resources. For manufacturing at the s-th enterprises-supplier the l-th kind of complete set’s unit it is needed to use the r-th kind of raw materials in the amount of a_{slr}, s = 1,2,..., S; l = 1,2,..., L_s; r = 1,2,..., R_s.

The initial inventory level of the r-th kind of raw material at the s-th supplier’s warehouse is q_{sr}^{(1)}. It is assumed that \(\sum_{r=1}^{R_s} q_{sr}^{(1)} \leq W_{1s}\), where \(W_{1s}\) is the warehouse’s capacity, \(s = 1,2,..., S\).

The all complete set manufactured by enterprises-suppliers are purchased by enterprise-manufacturer for manufacturing the K types of finished products. The capacity of warehouse for storage of the complete set is denoted by \(W_2\) and initial inventory level of each kind of complete set is q_{sl}^{(2)}, \(\sum_{s=1}^{S} \sum_{l=1}^{L_s} q_{sl}^{(2)} \leq W_2\).

Let a_{slk}^{(2)} be the amount of the l-th kind of complete set manufactured by the s-th supplier needed for manufacturing of the k-th type of finished product’s unit \(s = 1,2,..., S; l = 1,2,..., L_s; k = 1,2,..., K\).

The finished products come to the warehouse of manufacturer with the capacity \(W_3\) from which they must be delivered at the N points of destination (see Figure 1). The initial inventory level of the k-th type of finished product at warehouse is q_{k}^{(3)}, \(\sum_{k=1}^{K} q_{k}^{(3)} \leq W_3\).

Let \(d_{kn}\) be the total demand for the k-th type of finished product at the n-th destination over the planning horizon T. To avoid the trivial situation, we will assume that following conditions hold true:

\[ q_{k}^{(3)} < \sum_{n \in B_k} d_{kn}, B_k = \{n \mid d_{kn} > 0, n = 1,2,..., N\} k = 1,2,..., K. \]

These conditions have the obvious meaning: initial amount of each type of finished product doesn’t exceed the total demand for this product of the all destinations over the planning horizon.

In addition, we make the following assumptions:
- The market of raw materials is unlimited.
- All ordering of materials, complete set, and delivering the finished products occurs at the start of each period.
- The lead time is zero: that is, an order arrives as soon as it is placed.
- The production equipment of all enterprises is absolutely reliable.
- The capacities of production lines of all enterprises are limited only by capacities of warehouses’ for storage of raw materials and finished products.

Let us introduce the control variables:
- Let \(s_{sr}^{(1)}\) be the amount of the r-th kind of material ordered and purchased by the s-th enterprise-supplier in period t, for \(t = 1,2,..., T\).
Let $x_{slt}^{(2)}$ be the $l$th kind of complete set which the $s$th enterprise plans for output in the end of period $t$, for $t=1,2,...,T$.

Let $y_{kt}$ be the amount of the $k$th type of finished product planned for output in the end of period $t$, for $t=1,2,...,T$.

Let $z_{knt}$ be the amount of the $k$th type of finished product which is planned for delivery from warehouse to the $n$th destination in the end of period $t$, for $t=1,2,...,T$.

Let $I_{srt}^{(1)}$ be the inventory level of the $r$th kind of material at the warehouse of the $s$th supplier in the end of period $t$, for $t=1,2,...,T$.

Let $I_{slt}^{(2)}$ be the inventory level of the $l$th kind of complete set manufactured by the $s$th supplier in the end of period $t$, for $t=1,2,...,T$.

Let $I_{kt}^{(3)}$ be the inventory level of the $k$th type of finished product in the end of period $t$, for $t=1,2,...,T$.

Let $I_{srt}^{(2)}$, $I_{slt}^{(1)}$, $I_{slt}^{(2)}$, $I_{slt}^{(3)}$, $I_{slt}^{(4)}$, $I_{slt}^{(5)}$, $I_{srt}^{(1)}$, $I_{slt}^{(2)}$, $I_{slt}^{(3)}$, $I_{slt}^{(4)}$, $I_{slt}^{(5)}$ be the inventory levels of the $r$th kind of material, the $l$th kind of complete set, the $k$th type of finished product, the per unit order cost, the fixed order cost, the per unit production cost, and the cost of transportation of the $k$th type of finished product from manufacturer to the $n$th destination in period $t$, for $t=1,2,...,T$.

It is obvious that the following inventory-balanced equations are valid:

\[ I_{srt}^{(1)} = I_{srt}^{(1)} + \sum_{l=1}^{L_s} a_{srl}^{(1)} x_{slt}^{(2)}, \quad s=1,2,...,S; r=1,2,...,R_s, \quad t=1,2,...,T. \]  
(1)

\[ I_{slt}^{(2)} = I_{slt}^{(2)} + \sum_{k=1}^{K} a_{skl}^{(2)} y_{kt}, \quad s=1,2,...,S; l=1,2,...,L_s, \quad t=1,2,...,T. \]  
(2)

\[ I_{kt}^{(3)} = I_{kt}^{(3)} + \sum_{n\in B_k} z_{knt}, \quad k=1,2,...,K; t=1,2,...,T, \]  
(3)

where: $I_{srt}^{(1)} = q_{srt}^{(1)}$, $I_{slt}^{(2)} = q_{slt}^{(2)}$, $I_{kt}^{(3)} = q_{kt}^{(3)}$.

From (1)-(3), we obtain:

\[ I_{srt}^{(1)} = q_{srt}^{(1)} + \sum_{j=1}^{t} x_{srj}^{(1)} \sum_{l=1}^{L_s} a_{srl}^{(1)} x_{slj}^{(2)}, \quad s=1,2,...,S; r=1,2,...,R_s, \]  
(4)

\[ I_{slt}^{(2)} = q_{slt}^{(2)} + \sum_{j=1}^{t} x_{slj}^{(2)} \sum_{k=1}^{K} a_{skl}^{(2)} y_{kt}, \quad s=1,2,...,S; l=1,2,...,L_s, \]  
(5)
Since the total inventory levels
\[ \sum_{k=1}^{K} I_{k}^{(3)} = q_{k}^{(3)} + \sum_{j=1}^{t} y_{kj} - \sum_{j=1}^{t} \sum_{n \in B_k} z_{knj}, \quad k=1,2,\ldots,K; t=1,2,\ldots,T. \] (6)

On the other hand, the enterprises-suppliers for complete set manufacturing in period \( t \) can use inventories of materials which are at warehouses in the end of period \( t-1 \) only, that is
\[ \sum_{l=1}^{L_{s}} x_{slr} - \sum_{r=1}^{R_{s}} x_{sr} \leq W_{1s}, \quad s=1,2,\ldots,S; t=1,2,\ldots,T. \] (7)

Similarly, for enterprise-manufacturer the following restrictions must be fulfilled
\[ \sum_{k=1}^{K} y_{kj} - \sum_{l=1}^{L_{s}} x_{slk} \leq W_{2}, \quad t=1,2,\ldots,T. \] (8)

In period \( t \) it can’t be delivered the \( k \)th type of finished product at the all destinations in amount more than inventory level \( I_{k, t-1}^{(3)} \), therefore
\[ \sum_{n=1}^{N} z_{knt} \leq I_{k, t-1}^{(3)}, \quad k=1,2,\ldots,K; t=1,2,\ldots,T. \] (9)

From (10)-(12), taking into account the relations (4)-(6), we obtain
\[ \sum_{l=1}^{L_{s}} x_{slr} - \sum_{r=1}^{R_{s}} x_{sr} \leq W_{1s}, \quad s=1,2,\ldots,S; t=1,2,\ldots,T. \] (14)
\[ \sum_{j=1}^{t} y_{kj} - \sum_{l=1}^{L_{s}} x_{slr} \leq W_{2}, \quad t=1,2,\ldots,T. \] (15)
\[ \sum_{j=1}^{t} \sum_{l=1}^{L_{s}} x_{slk} \leq W_{3}, \quad t=1,2,\ldots,T. \] (16)

The conditions of non-negativity of control parameters must be added to restrictions (7)-(9), (13)-(16), i.e.
\[ x_{srt}, x_{slr}, y_{kt}, z_{knt} \geq 0, \forall s, l, k, r, n. \] (17)
As objective function we choose the total logistic cost for SC under consideration over the planning horizon. Taking into account the designations introduced previously, the expression for this total cost takes the form

\[
C = \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{r=1}^{R_s} \left[ c_{srt}^{(1)} x_{srt}^{(1)} + K_{srt}^{(1)} \delta_{srt} (x_{srt}^{(1)}) + h_{srt}^{(1)} (q_{srt}^{(1)}) + \sum_{j=1}^{t-1} x_{srt}^{(1)} - \sum_{j=1}^{t} \sum_{l=1}^{L_s} a_{srl}^{(1)} x_{srl}^{(1)} \right] + \\
+ \sum_{s=1}^{S} \sum_{l=1}^{L_s} \left[ (c_{slt}^{(2)} + c_{slt}^{(3)}) x_{slt}^{(2)} + K_{slt}^{(2)} \delta_{slt} (x_{slt}^{(2)}) + h_{slt}^{(2)} (q_{slt}^{(2)}) + \sum_{j=1}^{t-1} x_{slt}^{(2)} - \sum_{j=1}^{t} \sum_{k=1}^{K} a_{skl}^{(2)} y_{kj} \right] + \\
+ \sum_{k=1}^{K} \left[ c_{kt}^{(4)} y_{kt} + h_{kt}^{(3)} (q_{k}^{(3)}) + \sum_{j=1}^{t-1} z_{knj} + \sum_{j=1}^{t-1} \sum_{j=1}^{t} \sum_{j=1}^{t} z_{knj} \right] + \sum_{k=1}^{K} \sum_{k=1}^{K} c_{knt}^{(5)} \right],
\]

where \( \delta(x) = 1 \) if \( x > 0 \), \( \delta(0) = 0 \).

Thus, we can formulate the following optimization problem: it is needed to find out the variables \( x_{srt}^{(1)}, x_{slt}^{(2)}, y_{kt}, z_{knt} \) satisfying the constraints (7)-(9), (13)-(16) and minimizing the function (18). This optimization problem may be solved, for example, by dynamic programming algorithm [4] or by the method based on reduction of our optimization model to partly integer linear programming problem [5].

Note that other configurations of SC in the frame of VAT-classification may be analyzed on the basis of the Wagner-Whitin model by similar way.

4. OPTIMIZATION MODEL FOR RANDOM DEMAND OVER PLANNING HORIZON

Now we will assume that values \( d_{kn}(\omega) \) are the continuous mutually independent random variables with the given probability densities \( \varphi_{kn}(d) \). Here we will apply the approach proposed by Williams in the work [9]. Put

\[
u_{kn} = \sum_{t=1}^{T} z_{knt},
\]

where \( u_{kn} \) is total amount of the \( k \)th finished product planned for delivery to the \( n \)th destination before realization of random demand \( d_{kn}(\omega) \). In result of its realization, one of two risks may occur:

1. \( u_{kn} < d_{kn}(\omega) \), i.e. demand will not be met;
2. \( u_{kn} > d_{kn}(\omega) \), i.e. there is necessity of storage of the \( k \)th finished product’s surplus at the \( n \)th destination.

We assume that both risks belong to the destinations, i.e. the all manufactured finished products are sold.

Let \( \pi_{kn} \) be the penalty for the \( k \)th finished product’s deficit at the \( n \)th destination, and \( h_{kn}^{(4)} \) be the holding cost for storage per unit of the \( k \)th product at the \( n \)th destination. Then average total logistic cost over the planning horizon is:

\[
\overline{C} = C + \sum_{k=1}^{K} \sum_{l=1}^{N} \left[ \pi_{kn} \int_{0}^{\infty} (u_{kn} - w) \varphi_{kn}(w) dw + h_{kn}^{(4)} \left\{ \int_{0}^{\infty} (w - u_{kn}) \varphi_{kn}(w) dw \right\} \right],
\]

where \( C \) is defined by (18). The function \( \overline{C} \) is concave in respect to the variables \( u_{kn} \). Taking second derivative of \( \overline{C} \) with respect \( u_{kn} \) and applying the Leibnitz rule, we obtain:

\[
\frac{\partial^2 \overline{C}}{\partial u_{kn}^2} = - (\pi_{ik} + h_{kn}^{(4)}) \varphi_{kn} (u_{kn}).
\]
Since \( \pi_{kn} + h_{kn}^{(4)} > 0 \) by definition, the expression in the right-hand side of the last equality is non-positive. Hence, function (20) is concave.

Consequently, we arrive at the following concave optimization problem: it is required to minimize the function (20) under constraints (7)-(9), (13)-(16), (19).

5. NUMERICAL RESULTS

Consider the numerical illustration of above model for the case of given demand. For the relatively small numbers of suppliers, destinations, types of finished products, raw materials, and complete set calculations may be made with the help of Excel. Put \( T = 4, S = 2, L_1 = 2, L_2 = 1, R_1 = R_2 = 2, K = 2, N = 3. \) The total number of variables is 60. For the sake of simplicity we put \( k_{sr}^{(1)} = k_{sl}^{(2)} = 0, \forall s, r, l, t. \)

The rest initial data are given in the Table 1. The result of calculation is presented in the Table 2. (the rest variables equal to zero). The minimal value of total logistic cost is 30274.16.

Table 1. Initial data for calculation

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### Table 2. Result of calculation.

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</tbody>
</table>

### Conclusions

In this paper, we studied by means of inventory control theory the models of SC. The results obtained above show that our approach may successfully be applied for modeling and optimization of joint activity plans for all participants of SC under consideration. Moreover, this approach is suitable for modeling and analysis of various configurations of SC (e.g. belonging to the VAT-classification but not only). For practical applications of models proposed above the standard software may be used. As topics for further investigations in this direction may be pointed out the following:

a) modeling different classes of logistical nets which include many manufacturers, suppliers, traders, transport companies, etc.;

b) taking into account the feedback between retailers and suppliers, manufacturers, transport, etc.;

c) application of different ways for risk control arising in SC and in logistical nets.
Abstract

Article presents selected aspects of dynamic modeling for optimization of production and finished products distribution in supply chain. The aim of the paper is development and analysis of dynamic optimization model of the A type of supply chain. Optimization models for fixed and for random demand over planning horizon are given. Appropriate numerical experiments are discussed and relevant results are presented. Conclusions about means of inventory control theory for the models of supply chain. The results obtained show that presented approach may successfully be applied for modeling and optimization of joint activity plans for all participants of SC under consideration.

Key words: supply chain (SC), VAT-classification of SC, A-type of SC, given and random demand, joint plans optimization, dynamic optimization model.

Dynamiczny model optymalizacji produkcji i dystrybucji wyrobów gotowych w łańcuchu dostaw

Streszczenie

Artykuł prezentuje wybrane aspekty dynamicznego modelowania i optymalizacji produkcji i dystrybucji wyrobów gotowych w łańcuchu dostaw. Celem artykułu jest utworzenie i analiza dynamicznego modelu optymalizacyjnego typu A łańcucha dostaw. Modele optymalizacyjne dla stałych i losowych zamówień w określonym horyzoncie czasowym zostały zaprezentowane. Przedstawiono odpowiednie eksperymenty obliczeniowe i omówiono ich wyniki. Artykuł podsumowano rozważaniami na temat teorii zapasów w modelowaniu łańcuchów dostaw. Uzyskane wyniki pokazują, że przedstawione podejście może być z powodzeniem stosowane do modelowania i optymalizacji wspólnych planów działań wszystkich uczestników rozważanego łańcucha dostaw. Słowa kluczowe: łańcuch dostaw (SC), klasyfikacja VAT łańcuchów dostaw, łańcuch dostaw typu A, stałe i zmienne zapotrzebowanie, optymalizacja wspólnych planów, dynamiczny model optymalizacyjny.

LITERATURE