Criteria Parameters for Fluid Flow Simulation in Tight Reservoir Rocks

Introduction

Nowadays, academic and industry units are focused on unconventional resources, as shale gas and tight gas, exploration and production. A lot of efforts is put into the scientific and logistic aspects, e.g. reservoir parameters estimation, process of well site and water tanks for hydraulic fracturing preparations. For the scientific point of view, the main concern is connected with the porosity and permeability estimation, as a parameters deciding about the reservoir potential.

Combination of computed microtomography and fluid flow simulation approach leads to rock permeability estimation as a reservoir parameter deciding about the rock potential to fluid (hydrocarbons and/or formation water) flow in porous space [1, 4, 5, 6, 7, 8]. For the proper simulation run and results two aspects are needed to be considered: modelling approach and the flow regime. The Knudsen number classifies the modelling approach and Reynolds number – flow regime.

Criteria parameters in fluid flow analysis in porous media

One of the most important parameter in process preparation of fluid flow simulation is the Knudsen number. The Knudsen number $Kn$ should be consider as a condition which classifies the modelling approach. The value of Knudsen number represents the ratio of the molecular mean free path length $\lambda$ of analyzed gas to a representative physical length scale $l$:

$$Kn = \frac{\lambda}{l}$$

where:

$\lambda$ – mean free path (m),

$l$ – representative physical length scale (m).

The mean free path represents the possible distance of the particles traveling of analyzed gas as the effect of their collisions. The mean free path for selected gas is strongly depended on the gas pressure and should be estimated for a mean gas pressure inside the whole analyzed sample. The representative physical length scale, in case of fluid flow simulation in the pore space, is a value of the lowest channel diameter in analyzed sample.

The possible modelling approaches according to Knudsen number [2] are presented in Table 1.
Table 1. Modelling approaches according to Knudsen number

<table>
<thead>
<tr>
<th>Modelling approach</th>
<th>Knudsen number (Kn)</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuum flow</td>
<td>( Kn \leq 0.001 )</td>
<td>Boltzmann, Euler, no-slip Navier-Stokes equations</td>
</tr>
<tr>
<td>Slip flow</td>
<td>( 0.001 &lt; Kn &lt; 0.1 )</td>
<td>Boltzmann, slip Navier-Stokes equations</td>
</tr>
<tr>
<td>Transition flow</td>
<td>( 0.1 &lt; Kn &lt; 10 )</td>
<td>Boltzmann, Burnett equations</td>
</tr>
<tr>
<td>Free molecular flow</td>
<td>( 10 \leq Kn )</td>
<td>Boltzmann equation</td>
</tr>
</tbody>
</table>

The second important criteria parameter in fluid flow simulation is the Reynolds number, which is calculated by modifying the equation for porous media:

\[
Re_p = \frac{\rho v_f d}{\mu}
\]

where:

- \( d \) – grain diameter (m),
- \( \rho \) – fluid density (kg/m\(^3\)),
- \( v_f \) – superficial velocity (m/s),
- \( \mu \) – fluid dynamic viscosity (Pa·s).

Reynolds number is determined in order to check if the simulation may be performed in range of laminar, transition or turbulent flow conditions (Figure 1). The range of Reynolds numbers for porous media is not unique. Respective ranges of applicability of flow equations determine the correct flow regimes. In different papers it is possible to find the various limits for Reynolds number [4, 9]. The exemplary limitations for the range of Reynolds number for porous media presented in Figure 1 are presented in Table 2.

![Flow rate (superficial velocity) in porous media as a function of pressure gradient with characterized ranges for laminar, transition and turbulent flow [9]](image-url)
Table 2. The ranges of Reynolds number for porous media $Re_p$ to classify flow condition

<table>
<thead>
<tr>
<th>Flow condition</th>
<th>Pre-Darcy Flow</th>
<th>Darcy Flow</th>
<th>Forchheimer Flow</th>
<th>Turbulent Flow</th>
</tr>
</thead>
</table>

**Method of criteria parameters estimation**

Computed microtomography is a powerful tool in obtaining 3D images of the rock pore space [10, 11]. The full analysis of the pore throats and channels may be carried out owing to micro-CT 3D images. To calculate the Knudsen number the representative physical length scale (diameter) in the smallest pore channel is required. These parameter may be measured on the basis of micro-CT 3D images. The Reynolds number is obtained using grain diameter which also may be estimated from micro-CT slices. After determination of criteria parameters for fluid flow simulation the proper modelling approaches and fluid regimes are set. The Knudsen number and Reynolds number were calculated for the Carboniferous sandstone sample, cored at 3154 m depth, of total porosity obtained from micro-CT equal to 12.6%. The Figure 2 presents the results obtained using the micro-CT technique.

![Fig. 2. The example of pore space visualization using micro-CT [10]](image)

On the basis of micro-CT images the values of representative physical length scale and grain diameter was determined. These values are required to estimate criteria parameters described above as a Knudsen and Reynolds numbers. To calculate the Knudsen number, value of representative physical length was founded from the 3D microtomographic image. The smallest pore channel diameter presented in Figure 3 is equal to 12 µm. Assuming the mean free path $\lambda$ of nitrogen, in standard temperature and pressure conditions, as equal to $10*10^{-8}$ m [3], the Knudsen number reaches value equal to $Kn=0.0008$. The Knudsen number reached value close but less than $Kn<0.001$, what allows to use the no-slip Navier-Stokes equations as the proper approach for the fluid flow simulation. If $Kn$ reaches higher value, then slip Navier-Stokes equations should be applied to the fluid flow modelling.
In case of Reynolds number estimation, the mean grain diameter value of rock sample was calculated based on similar approach as the Knudsen number. The average mean diameter for analyzed sample is equal to 250 μm. The fluid properties of nitrogen, used as in flow simulation were set as \( \rho_n = f(p_n, T_n) \), where \( p_n = 100 \) kPa and \( T_n = 273.15 \)K, \( \rho_n = 1.123 \) kg/m\(^3\) and \( \mu_n = 1.663 \times 10^{-5} \) Pa*s. If nitrogen superficial velocity \( v_f \) through the pore space is equal to 0.05 m/s (based on measured nitrogen mass flow rate and its density) the Reynolds number, calculated using equation (2), is equal to 0.84 and confirms the laminar flow condition in pore space, in Pre-Darcy range.

**Conclusions**

The results of computed microtomography for the sandstone sample were discussed. On the basis of micro-CT images the representative physical length diameter in the smallest pore channel and grain diameter were obtained by the detailed analysis of the 3D images. For carrying out the correct fluid flow simulations two criteria parameters have to be evaluated: the Knudsen number and Reynolds number. The Knudsen number determines the modelling approach: continuum, slip, transition or free molecular. Reynolds number checks if the simulation may be performed in laminar flow conditions (Darcy law regime). 3D image analysis allowed to qualitative and quantitative estimation of the Carboniferous sandstone sample pore throats and channels and informed about the sample pore space complexity. Computed microtomography together with fluid flow simulations gave a new insight in determination of the rock reservoir parameters. Analysis of micro-CT images provides the information about pore space complexity.

**Abstract**

Combination of computed microtomography and fluid flow simulation approach leads to rock permeability estimation as a reservoir parameter deciding about the rock potential to fluid (hydrocarbons and/or formation water) flow in porous space. For the proper simulation run and results two aspects are needed to be considered: modelling approach and the flow regime. The Knudsen number classifies the modelling approach and Reynolds number – flow regime. The Knudsen number and Reynolds number were calculated for the Carboniferous sandstone sample, cored at 3154 m depth and with total porosity obtained from micro-CT equal to 12.6%.

**Keywords:** Knudsen number, Reynolds number, computational fluid dynamics, computed microtomography, reservoir potential of rocks
PRZEDSTAWIENIE

W pracy zostały omówione parametry kryterialne oraz sposoby ich wyznaczania w celu poprawnego modelowania zjawiska przepływu płynu przez ośrodki porowate z zastosowaniem komputerowej mechaniki płynów. Poprawny dobór poszczególnych równań i modeli podczas komputerowej symulacji przepływu płynu wymaga obliczenia kryterialnych parametrów, jakimi są liczba Knudsena i liczba Reynoldsa. Wartości liczb kryterialnych determinują możliwe do zastosowania podejście w modelowaniu przepływów oraz określają zakres charakteru przepływu, jaki występuje w analizowanym przypadku. W pracy przedstawione zostały wyniki obliczeń parametrów kryterialnych dla piaskowca karbońskiego.

Słowa kluczowe: liczba Knudsena, liczba Reynoldsa, modelowania zjawiska przepływu płynu, ośrodek porowaty

References


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