Theoretical Evaluation of Vehicle Stability in Mountainous Areas

INTRODUCTION

Every year vehicle manufacturers give more attention to the safety of drivers and passengers. Even though the safety might be influenced by a human factor, the passive and active systems of the vehicle itself play a major role. Construction design of vehicle components is attributed to the passive safety systems. As a rule these passive systems do not change during exploitation, meanwhile the environment creates varying conditions influencing the stability of the vehicle.

Analysis of various accidents of Toyota Land Cruiser 100 (LC) vehicles with a diesel engine 1HD–FTE used in Ghor region in Afghanistan by the Lithuanian army during a period from Summer 2005 to the end of 2011 shows irreversible losses of 15% of the vehicles [1]. An official inquiry was carried out after each incident, and the main reasons of the losses were found to be complicated environmental conditions – uphill and downhill slopes of the roads with a constant horizontal curving of varying radius with plenty of sharp turns as small as 10 m in radius and 180° of rotation (Figure 1).

Elevation of Afghanistan roads can reach hundreds and thousands of meters above the sea level, and sharp changes in the elevation are typical. The mountainous roads are classified by the intensity of the changes in elevation and the degree of curving. Two categories are differentiated: hilly roads and mountain-pass roads. The hilly roads have a varying cross-section due to rapid changes in upward and downward slopes. Mountain-pass roads have a rather uniform cross-section, long downhill and uphill slopes. Complex relief of the mountain roads creates adverse conditions for vehicle exploitation described by a large total resistance to the vehicle movement and complicated vehicle control conditions. Weather conditions also play an important role, because most of the investigated accidents (65%) happened during winter and spring [1].

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Object of investigation – specially upgraded Toyota Land Cruiser 100 vehicles used in complicated (mountainous) conditions. Work aim – determine the stability limits of the vehicles under investigation.

1. METHODOLOGY

Theoretical stability calculations of Toyota Land Cruiser 100 vehicles are carried out using the methodology of prof. A. Tursumov [6]. Forces acting on a vehicle in mountains are illustrated in Fig. 2 and Fig. 3. Vehicle parameters necessary for calculations are either obtained from its technical specification or derived. The center of gravity was determined using Janulevičius' methods [2], because the vehicles under question have been upgraded with armored side- and back-door plates and a large luggage compartment on top. Therefore, the center of gravity is considerably higher for these vehicles than for conventional models. The parameters are summarized in Table 1.

Table 1. Main parameters of the analyzed vehicle used in calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M, total mass</td>
<td>4120 kg</td>
<td>h, center of mass</td>
<td>1117 mm</td>
<td>rw, wheel radius</td>
<td>395 mm</td>
</tr>
<tr>
<td>unamortized mass</td>
<td>168 kg</td>
<td>l1, 1st axis track width</td>
<td>1640 mm</td>
<td>hpi, center of amortized</td>
<td>1193 mm</td>
</tr>
<tr>
<td>mass</td>
<td></td>
<td>mass</td>
<td></td>
<td>mass</td>
<td></td>
</tr>
<tr>
<td>C1, 1st axle spring stiffness</td>
<td>900 N/cm</td>
<td>l2, 2nd axis track width</td>
<td>1635 mm</td>
<td>r, tie rod length</td>
<td>150 mm</td>
</tr>
<tr>
<td>C2, 2nd axle spring stiffness</td>
<td>1000 N/cm</td>
<td>a, distance from</td>
<td>1363 mm</td>
<td>k1, half the distance</td>
<td>850 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st axis to center of mass</td>
<td></td>
<td>between the axes of the</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>king pins</td>
<td></td>
</tr>
<tr>
<td>C_f, friction</td>
<td>0.3</td>
<td>L, wheel base</td>
<td>2850 mm</td>
<td>h1, upper attachment</td>
<td>600 mm</td>
</tr>
</tbody>
</table>
When a vehicle is climbing uphill, its weight and driving momentum create reaction forces on its wheels. Stability of vehicles depends on the perpendicular (to the road) reaction forces. The stability is lost if the perpendicular reaction force acting on the front wheels $R_{Z1}$ becomes equal to zero. The perpendicular reaction forces are given by

$$R_{Z1}(\alpha) = G \cdot (b \cdot \cos \alpha - h \cdot \sin \alpha)/L, \quad R_{Z2}(\alpha) = G \cdot (a \cdot \cos \alpha + h \cdot \sin \alpha)/L$$

where $R_{Z1}$ and $R_{Z2}$ are the perpendicular reaction forces acting on the front and back wheels correspondingly, $G$ is the total weight of the vehicle, $a$ - distance along the road from the front axle to the center of gravity, $b$ - distance along the road from the back axle to the center of gravity, $h$ - height of the center of mass above the road, $L$ - wheel base, and $\alpha$ - the angle of the upward slope of the road.

It is quite common for a mountainous road to have a cant (a cross slope). Forces, acting on a vehicle making a turn of a radius $R$ on a canted road of cross slope angle $\beta$ are shown in Figure 3. If the vehicle is making a left turn, then the centrifugal force $F_c$ decreases the load on the left wheel. The centrifugal force $F_{c1}$ acting on the front axis is

$$F_{c1} = \frac{R_{Z1}(\alpha)}{g} \cdot \frac{v^2}{R}, \quad (2)$$

where $g$ is the acceleration due to gravity. Noting that the initial load on the front axis is $G_I = R_{Z1}(\alpha)$ and summing up the force moments around the bottom point of the right front wheel gives the load on the front left wheel:

$$R_{Z1L}(\alpha, \beta) = \frac{1}{l_1} \cdot \left\{ (R_{Z1}(\alpha) \cdot \cos \beta + F_{c1} \cdot \sin \beta) \cdot \frac{l_1}{2} - (F_{c1} \cdot \cos \beta - R_{Z1}(\alpha) \cdot \sin \beta) \cdot h \right\} \quad (3)$$

where $l_1$ is the track width of the front wheels. Similarly the right front wheel load is

$$R_{Z1R}(\alpha, \beta) = \frac{1}{l_1} \cdot \left\{ (R_{Z1}(\alpha) \cdot \cos \beta + F_{c1} \cdot \sin \beta) \cdot \frac{l_1}{2} + (F_{c1} \cdot \cos \beta - R_{Z1}(\alpha) \cdot \sin \beta) \cdot h \right\} \quad (4)$$

Assuming a cornering force $Y$ acting on a wheel is proportional to its perpendicular reaction force, and summing the forces acting along the axis, we obtain the cornering force for the left and right wheel correspondingly:

$$Y_{1L}(\alpha, \beta) = \frac{R_{Z1L}(\alpha, \beta)}{R_{Z1L}(\alpha, \beta) + R_{Z1R}(\alpha, \beta)} \cdot (F_{c1} \cdot \cos \beta - R_{Z1}(\alpha) \cdot \sin \beta), \quad (5)$$

$$Y_{1R}(\alpha, \beta) = \frac{R_{Z1R}(\alpha, \beta)}{R_{Z1L}(\alpha, \beta) + R_{Z1R}(\alpha, \beta)} \cdot (F_{c1} \cdot \cos \beta - R_{Z1}(\alpha) \cdot \sin \beta).$$
Fig 3. Forces acting on front wheels of a vehicle making a left turn of radius $R$ on a sideways sloped road (rear view)

Having the values of both the cornering force $Y$ and the perpendicular reaction force $R_z$ it becomes possible to obtain the slip angles $\delta$ for the tires. For this purpose we use Pacejka’s Magical Formula [7]:

$$Y = D \sin[C \tan(B \cdot \delta - E \cdot (B \cdot \delta - \tan(B \cdot \delta)))],$$

$$C = a_0, \quad D = a_1 R_z^2 + a_2 R_z, \quad E = a_3 R_z^2 + a_4 R_z + a_5,$$

$$B = a_6 \sin(a_4 \tan(a_5 R_z)) / (C \cdot D)$$

where empirical coefficients are used: $a_0 = 1.3, a_1 = -22.1, a_2 = 1011, a_3 = 1078, a_4 = 1.82, a_5 = 0.208, a_6 = 0, a_7 = -0.354, a_8 = 0.707.$

The slip angle $\delta$ is not expressed explicitly in the formula. However, it is possible to calculate it numerically for any given combination of $Y$ and $R_z$ to obtain the dependence of the slip angle $\delta$ on the uphill slope $\alpha$, road cant $\beta$, vehicle turn radius $R$ and velocity $v$.

In particular, we are interested in the difference between the slip angles of the first and second axle: $\Delta \delta = \delta_1 - \delta_2$. Whenever $\Delta \delta > 0$, we have under-steering conditions – the vehicle tends to go straight instead of turning and the driver needs to compensate this angle by turning the wheels somewhat more. The opposite situation – over-steering happens when $\Delta \delta < 0$. This is more dangerous as the back of the car turns more than the front, and the vehicle becomes prone to rolling over.

There is one more component we need to take into account when discussing the sideslip angle of a turning vehicle – it is the additional angle $\theta$ the driver needs to turn the wheels due to interaction of the suspension and vehicle’s steering mechanism. When the vehicle is making a turn, the centrifugal force makes the amortized body of the vehicle rotate sideways by an angle $\psi$. Redistribution of normal forces acting on wheels deforms the springs of MacPherson suspension differently on different sides of the car, the steering knuckle of the king pin changes its direction by an angle $\gamma$, the axis of the king pin itself moves by a distance $S$ and the wheel turns by an angle $\theta$ independently of the driver.

When a car moves in a curve of radius $R$, the deformation of the first axle’s spring under a static load can be given as [6]:

$$h_0 = \frac{G_{1L} - m_{na}g \cdot 0.5}{C_1},$$

where $m_{na}$ – unamortized mass of the front axle of the vehicle, $C_1$ – stiffness of the spring, and

$$G_{1L} = \frac{G_1}{2} = \frac{R_{21}(\alpha=0)}{2}.$$  

The change of the deformation of the spring due to a change in the load is then

$$\Delta h_1 = (R_{21L}(\alpha, \beta) - G_{1L})/C_1.$$  

(7)
According to [6], the distance the king pin axis moves can be expressed as:

\[ S = k_1 \cdot \tan \gamma = k_1 \cdot \tan(\Psi \cdot \tan \varphi) \]  \hspace{1cm} (10)

where

\[ \tan \varphi = \frac{h_0}{0.75l_1} \]  \hspace{1cm} (11)

where \( k_1 \) is half the distance between the axes of the king pins, \( l_1 \) is the distance between the king pin axis and the point of attachment of the spring, and \( h_0^* = h_0 + \Delta h_1 \). The angle of the tilt of the amortized masses of the vehicle is given by [6]:

\[ \Psi = \frac{F_{ch_s}}{2 \left( C_{p1} + C_{p2}\right) d_s^2 - G_{na} h_s} \]  \hspace{1cm} (12)

where \( G_{na} \) – unamortized weight, \( C_{jp} \) and \( C_{2p} \) – stiffness of springs on the first and second axle, \( d_s \) - spring length, and \( h_s \) - radius of the tilt of the car’s center of mass, given by:

\[ h_s = h_a - h_1 \]  \hspace{1cm} (13)

where \( h_a \) – height of center of gravity of amortized mass, \( h_1 \) – height of the upper attachment point of the spring from the ground. Knowing the height of the center of gravity of the whole vehicle \( h \), the amortized weight \( G_{am} \) and the wheel radius \( r_w \) we can obtain

\[ h_a = h + \frac{G_{na}}{g_{am}}(h - r_w) \]  \hspace{1cm} (14)

Finally, we express the angle \( \theta \) as

\[ \tan \theta \equiv \theta = \frac{s_1}{r} \]  \hspace{1cm} (15)

where \( r \) is the length of the tie rod of the steering gear.

Now the total sideslip angle of the left front wheel of a vehicle turning on a sloped road can be given as

\[ \delta_{1L}^T = \delta_{1L} + \theta \]  \hspace{1cm} (16)

where \( \delta_{1L} \) depends on the redistributed reaction forces due to the road slope, and \( \theta \) arises due to the specifics of interaction between the steering mechanism and the suspension. Further section analyzes the difference between the sideslip angles of the left wheels of the first and second axles:

\[ \Delta = \delta_{1L}^T - \delta_{2L} = \delta_{1L} - \delta_{2L} + \theta \]  \hspace{1cm} (17)

RESULTS

First we consider the situation without the influence of the steering mechanism (\( \theta = 0 \)). The difference \( \Delta_\theta = \delta_1 - \delta_2 \), between the sideslip angles of the left wheels of the first and second axles of reinforced Toyota Land Cruiser 100 vehicles driving at a speed of \( v = 20 \) km/h and making a turn of radius \( R \) on an upward sloping road of angle \( \alpha \) is presented in Figure 4. The sideways slope of the road is \( \beta = 0 \). Negative values of \( \alpha \) denote the downhill slope, \( R > 0 \) denotes a left turn, and \( R < 0 \) means a right turn. The left wheels of the vehicle going uphill (\( \alpha > 0 \)) and turning left (\( R > 0 \)) will experience a slight over-steering (\( \Delta_\theta < 0 \)), while somewhat more pronounced under-steering occurs when going downhill. The tendency increases the turn becomes sharper. At an upward slope slightly below \( \alpha_0 \approx 4 \) degrees the slip angle of the front and back left wheels equalizes, because the center of gravity shifts backwards and equalizes the load on the front and back axle. In fact, the exact angle can be obtained from the equation \( R_{\alpha_0}(\alpha) = R_{\alpha_0}(\alpha) \). Using (1) we obtain

\[ \alpha_0 = \arctan \left( \frac{b-a}{2h} \right) = 3.18 \]  \hspace{1cm} (18)
When we consider the full difference of the sideslip angles $\Delta$ defined by (17), the instability becomes prominent for larger road slopes, see Figure 5. A significant oversteer ($\Delta < 0$) occurs for the vehicle going downhill ($\alpha < 0$) and turning right ($R < 0$), when the cross-section of the road unfavorably declines to the left by $\beta = 4$ degrees.

Even though the oversteer can turn the vehicle sideways, it is not large enough to be problematic if the radius of the road turn is larger than 30 m. However, considering the small friction of the mountainous roads (especially in winter and spring conditions) with friction coefficient $C_f = 0.3$ or...
even smaller, another more prominent problem becomes evident. When the vehicle takes a turn at a higher speed, the centrifugal force $F_c$ can easily overcome the wheel friction. If condition $F_c > C_f R_z$ is fulfilled for at least one axle then the vehicle starts sliding off the road. Here $R_z$ is the load of the wheel, e.g. (3). Indeed, the calculations show a very narrow range of allowed speeds, Table 2, even on a flat road. The allowed speed for taking a turn decreases even more if the road is sloped and canted.

**Table 2.** Matrix of allowed speeds for different road turn radii before the vehicle starts sliding (the road is flat, $\alpha = 0$ and $\beta = 0$).

<table>
<thead>
<tr>
<th>Radius, m</th>
<th>V, km/h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>6.3</td>
<td>slide</td>
</tr>
<tr>
<td>10</td>
<td>slide</td>
</tr>
<tr>
<td>20</td>
<td>slide</td>
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<tr>
<td>30</td>
<td>slide</td>
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<tr>
<td>40</td>
<td>slide</td>
</tr>
<tr>
<td>50</td>
<td>slide</td>
</tr>
<tr>
<td>60</td>
<td>slide</td>
</tr>
</tbody>
</table>

It is important to note the center of mass of the analyzed vehicles ($h = 1117$ mm) was shifted up considerably during their reinforcement. Calculating the static stability factor

$$ S_s = \frac{T}{2h} $$  \hspace{1cm} (19)

where $T$ is the track width, we obtain a very small value $S_s = 0.73$. According to the extensive statistical study of vehicle accidents by Mengert [4], the probability $p_R$ of rollover versus non-rollover for a car involved in an accident is mostly determined by the static stability factor [5], and can be given as

$$ p_R = \frac{100}{(1+S_s^2)} $$ \hspace{1cm} (20)

which reaches a high $p_R = 89.4\%$ in our case. Consequently, when the vehicle slides off the road it is almost certain to rollover.

**CONCLUSIONS**

Analysis of stability of reinforced Toyota Land Cruiser 100 vehicles used by the Lithuanian army in Afghanistan mountains indicates that:

- under-steering and over-steering situations can arise due to different sideslip angles of front and back axle wheels when driving on a sloped road, but the steering stability is sufficient if the driving speed does not exceed 30 km/h for turns sharper than 30 m in radius;
- a larger problem is the low friction of dirt roads, and the calculated allowed speeds are quite low in order to eliminate the possibility of sliding;
- if the allowed speed is exceeded, and the car under question slides off the road, its probability to rollover is unusually high (89.4%) due to overly elevated center of mass. The situation could be somewhat improved by eliminating the top luggage compartment.

**Abstract**

A probability for a vehicle to overturn when climbing a hill or when going straight downhill is rather small because the design of vehicles makes them sufficiently stable with respect to a longitudinal bank. When the vehicle is loaded and moving uphill, its rear (driving) wheels receive a substantial additional load, while the load on the front wheels decreases. Consequently, the decreased traction of the front wheels diminishes the control of the vehicle, and it can also become completely uncontrollable. The article investigates the influence of a mountainous terrain to the controllability of vehicles. Situations commonly arising during exploitation of vehicles in mountainous areas are analyzed. Theoretical calculations of sideslip angles are illustrated by examples of recent accidents of military patrol cars in Afghanistan. Realistic estimates of safe driving speeds.
are presented. The main reason for the high rate of accidents is found to be an overly high center of mass of reinforced vehicles under consideration.

BIBLIOGRAPHY