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VRP-R: Robust routing problems solutions based on the HILS platform

INTRODUCTION

The proposal of HILS (Hierarchical Integrated Intelligent Logistic Systems) platform for development of ILS systems was proposed from the perspective of capabilities of new available technologies and professional tools [12] [14] [16]. The main specifications of HILS offering features includes: practically efficient individual ILS systems solutions, dedicated to high level of complexity, stimulating the system efficiency and productivity (e.g. activities coherency and understanding system processes/mechanisms), exploration of system-wide communication and integrated functionalities, flexibility/intelligence, new ILS activity supporting technologies, vehicles/digital maps platforms. This platform is a crucial step enabling the development of practically efficient ILS systems proposals dedicated to very complex dynamic, stochastic and behavioural interactions existing in logistics processes [11-16]. The real time system-wide identification, intelligent diagnosis, estimation and prediction of these interactions, conditioning the efficiency and productivity of the crucial ILS systems integrated functionalities (e.g. management, adaptation, routing/scheduling, surveillance and monitoring and direct control actions). These functionalities are realized in integrated way by the hierarchical multi-layer functional structure and are characterized by different tasks specifications (e.g. decisions time horizons, types of processes representations and optimization problems, reaction times to real time recognized and diagnosed events etc.). This platform is embedded in a nowadays available advanced sensing, information, computer, communication enabling technologies supported by capabilities of vehicle platforms (e.g. vehicle navigation, location, v-v. v-i communication, vehicle-probe etc.). In addition it is supported by professional exploration of integration (co-operative complex systems approach with multi: networks/layers/users/services/ objectives specifications) and intelligence (recognizing, diagnosing and understanding complex interactions and behavioral patterns, decreasing uncertainty, unpredictability, recognizing the abnormal traffic events and opportunities for very efficient actions). Five Layers HILS in natural way integrate and vertically orders (in time, frequencies of interventions, aggregation levels) wide spectrum of decision making and optimal control functions that additionally are supported by integrated data, knowledge and tools basis equipped with dedicated DSS and CASD. At the upper layer the logistic strategy is created by multi-criteria approach integrating both layers tasks. The management actions concerning the flows of materials, means, information in the areas of supply, production, distribution from the point of view of clients are realized. The integrated production management consists of several stages: Activity Targets Establishing→ Demand Prognosis→ Organization of Production Means → Decisions Optimization → Costs Analysis. The general co-operative HIILS multi-layers operation may be presented by the following cycle [5-7] [14-15]:

1. The Coordination and Management layer offers ALIS (Advanced Logistics Information Service) concerning SupNet interactions, essential events (e.g. incidents, critical network elements), network state specifications, global preferences and constraints and coordination premises.
2. Adaptation layer offers Dynamic Network Updating: network structure (available elements), routes (patterns, nodes/links and specifications), levels of congestion (incidents, available throughput).
3. Optimisation layer solves different types of VRP (Vehicle Routing and Scheduling) problems after robust estimation of the routing problem specifications e.g. travel times from depot to customers.

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4. Supervision layer: In this layer the real-time monitoring of the logistic system environment is realized as well as the monitoring of parameters of the system (operational efficiency of the system resources, system reliability, shortages, costs, demands). The modern multi-media technologies are used for ALIS (Advanced Logistic Information Service), visualization, warning and alarm generation purposes. In consequence wide spectrum of professional anticipative and preventive actions practically on all layers of the proposed system can be realized.

5. Control Layer: Very important new functional element at the bottom direct control layer concerns the full integration of the tasks of intelligent supervision with the intelligent adaptive control actions (see Fig. 1-3) realized by the DISCON and PIACON control methods [8]. The practical proposals of traffic multi-criteria control capabilities realized in hierarchical multi-layer adaptive, optimisation and direct control structure were presented in [1-3] [7-9] [13]. In this context very important new adaptive layer tasks are concerned with RG, RGLC, AIDM of vehicle route guidance in the network, route guidance to logistic centres and automatic incidents detection and management functions respectively.

1 ROUTING PROBLEMS BASED ON HILS PLATFORM

Vehicle routing problems are a well serviced by the HILS platform. To illustrate of HILS application, the VRP-TW (Vehicle Routing Problem with Time Windows) is presented. In this example the HILS upper layers interactions offer the context related updated demand and network specifications (i.e. adequate level of service and routing parameters values for routing optimisation layer supported by intelligent monitoring and supervision layer activity. At the bottom direct control layer the DISCON dispatching control method was adopted to the real-time routing disturbances compensations with routing treated as reference schedule to be stabilized by dispatching control actions. The formal VRP typical routing problem specifications implemented in the optimisation layer are as follows [13-15]:

1. Network: $G=(V,L)$ with customers located in the nodes $v \in V$ (#v=n) and L-links (#L=m)

2. Nodes: $\forall i \in V: \{d_i, p_i, TW=[ts_i, te_i], t_i, t^{i_1}\}$ represents respectively: demand forward/backward, time-windows, arrival/service times at customer i-th.

3. Links: $\forall (i, j) \in L: \{T_{ij}, D_{ij}, C_{ij}, R_{ij}\}$ represents travel times/distances, cost, uncertainties

4. Routes: originating/terminating at one/multi depots, homogeneous/heterogeneous depots

5. Service specifications: (SP1: $\forall i$: must be assigned to exactly one/several routes/vehicles ; SP2: $\forall i$: is visited only ones/several times; SP3: demand all customers are serviced)

6. Fleet of vehicles: homogeneous/heterogeneous; vehicle types related capacity

7. Customers: fixed/elastic demand; known delivery/pick up demands

8. Adopted objectives: total distance /travel time, costs, LoS-measures, sum of lateness at customers

The typical VRP-TW problem at HILS platform can be formulated as follows: selection of types of ADV admissible decision variables: $x_{ijk} \in \{0,1\}$ binary variables used to assignment of the network arc (i, j) to the route of the k-th vehicle; $z_{ij} \in R_1^+$ demand delivered to customers routed after node “i” i.e.

(ADV): $\{x_{ijk} \in \{0,1\}; \quad z_{ij} \in R_1^+\}$

Flow conservation principles for delivery demands and admissible routing flows:

$$\sum_{i=1}^{N+1} z_{ij} - \sum_{j=1}^{N+1} z_{ji} = d_j \quad (FCA)$$

$I\!$S upper layers LoS specifications: all customers must be visited only once and assigned to exactly one route from finite set of routes/vehicles. Customers Visits Specifications are:

$\sum_{i=1}^{N+1} \sum_{k=1}^{K} x_{ijk} = 1 \quad (CVS)$

Operational Specifications: travel times representation, service time windows functionalities, customers visiting times sequence, maximum admissible routes distances/ travel times are:
The identical fleet of vehicles $k=1,...,K$ specifications $\forall k \in K$: \{limited capacity $c_{\text{ap}} = \text{cap}$. The VRP-TW optimization problem may be formulated as follows (see Fig.1):

$$Q = \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{n} c_{ij} \cdot x_{ijk} \leq L$$

**OPTIMIZATION: ROUTING option LAYER 3**
- Optimization problems: multi-criteria multi-modes junctions co-ordination/ zones synchronization, dedicated intelligent optimization of critical network elements
- Intelligent routing optimization: multi-criteria, robust, different options
- Estimation of area-based traffic routing parameters and traffic situation markers
- Reference routing specification selection, updating and trade-offs

**INTELLIGENT SUPERVISION / MONITORING LAYER 2**
Intelligent supervision: traffic processes, incidents, behavioral reactions

**DIRECT MULTI-CRITERIA CONTROL LAYER 1**
- DISCON-PIACON
- DISCON-ROBUST
- PIACON-ROBUST

Fig.1 Illustration of the vehicle routing dedicated HILS bottom layers functional co-operation

Optimal dispatching control for VRP-TW solutions realized at the bottom direct control layer (see Fig. 2-3) is dedicated to compensate off-reference trajectory (determined in optimization layer by routes selection) deviations and essential increasing the robustness of the actual obligatory trajectory.[18]

Dispatching control actions dynamically evolving in 2-D (time and space) are integrated in DISCON method [13] in an optimal dynamic control strategy resulting from the minimization of some selected measures of service standards e.g. off-reference routing trajectory deviations. Wide spectrum of DISCON control tasks (punctuality, regularity, synchronizing priority control) call for a multi-criteria integrated approach. In the papers ([1][7-9][13]) the 1-D and 2-D (primal and dual) dynamic control plant representations have been developed and illustrated by a family of single criteria optimal control DISCON solutions of dead-beat, LQ, LQG type. The efficient dispatching multi-criteria priority control mode at traffic signalized intersections was proposed as an option in the PIACON [8-9]. For this, dynamic control models representing evolution of the vehicle off-reference trajectory deviations were equipped with real-time parameters estimation procedures. After the detection of the vehicle arrival to the intersection and evaluation of its measure of deviations the dynamic trade-offs with conflicting individual traffic and public transport demands are established by PIACON and appropriate priority control options for DISCON was proposed. The PIACON generated “reference trajectory” robust priority control option was proposed [9] (see Fig.3). The development of the multi-criteria control method with directly updated service quality standards (i.e. admissible levels of the stochastic measures of deviations) expressed in terms of the criteria to be minimized is a new contribution to the robust activities presented in this paper. The special emphasis has been devoted to
multi-criteria robustness features of the dispatching control actions. In general, 2-D state-space vehicle trajectories based model [9] consist of state vector \( x(i,j) \) with coordinates \( i \in I \) integer-valued vehicle index „” and \( j \in J \) index possible service-oriented points along the route (e.g. customers, traffic intersections). The input \( u(i,j) \) and output \( y(i,j) \) vectors are connected with states through real matrices of appropriate dimensions and create state-space equations in the form with state vector to be

\[
\begin{align*}
x(i+1, j+1) &= \sum_{l=0}^{j} A_{ij} x(i+l(l-1)/2, j+l(l-1)/2) + B_{ij} u(i+l(l-1)/2, j+l(l-1)/2) + \sum_{l=0}^{j} W_{ij} z_{ij} - \sum_{l=0}^{j} W_{ij} \delta_{ij} \\
y(i, j) &= C_{ij} x(i, j)
\end{align*}
\]

the measure of deviation between planned and actual vehicle trajectories. The 1-D dynamical system representations: In this representation the planned and actual vehicle trajectories are the functions of 1-D domain. Assuming for a given time window TW the reference point of vehicle arrival the corresponding punctuality control model has the form: \( x_j+1=x_j+u_j+z_j \) where \( i \in I \) vehicle/j \( j \in J \) of possible control points along the route indices; \( x_{ij} \) -state variables representing off-reference deviations; \( u_{ij} \) -control and \( z_{ij} \) -disturbance variables (off-schedule deviations in travel times and driver behavior). The optimal control problem: the appropriate elimination of the off-reference trajectory deviations of the vehicles on route represented by some control model may be formulated as:

\[
J_{T-j} = \sum_{i=0}^{T-1} \|x_i\|_Q^2 + \sum_{i=0}^{T-1} \|x_i\|_R^2 + \|u_{k} - u_{k-1}\|_S^2
\]

where \( Q_k, R_k, \) and \( S_k \) are weighting matrices such that \( R_k, S_k \) are positive definite. The first and the second term may be regarded as the off-reference deviations penalties at point (T) and all precedent vehicle stops. The last two terms penalize both the weighted sum of squares of control actions and the weighted sum of squares of the first backward differences in control actions. These terms represent the costs of the magnitude of control actions and adaptive control smoother features. The state and controls may be additionally bounded by hard inequality constraints (CON): \( x_k \in TW_k=[L_k,U_k] \) or imposed by PIACON methods: local reference trajectory \( x_k = reftraj_k \), and control \( u_k \in [U,V] \).

The optimal control problem may be stated as [9]: \( PO \min \ u(k) \ k=1,..,T \ JT-j \ (CON) \).

Reference trajectory priority control is similar to offered by DISCON-PIACON [8] interaction for public transport. After the detection of the vehicle arrival to the junction and evaluation of its measure of deviations the dynamic trade-offs with conflicting traffic and robust criteria (expressed in terms of norms of the Hardy spaces \( H_2 \) and \( H_\infty \) ) demands are established by PIACON and appropriate robust local reference priority options called the “local reference point/trajecory” for DISCON are proposed (see Fig.3).

**Fig.2 Intelligent co-operative multi-criteria logistics priority control at bottom control layer**
2 ROBUST CAPACITATED VEHICLE ROUTING PROBLEM WITH TIME WINDOWS

In the most Capacitated Vehicle Routing Problems with Time Windows (CVRPTW) the typical assumption concerns the full incidence matrix between graph nodes. Because the real transportation networks not fulfill this assumption (i.e. not all pairs of nodes are connected with others) in this paper the proposal of integration of SPP (Shortest Path Problem) with CVRPTW problem was presented.

The robust features on uncertain network arcs travel velocities was realized by calculation of the robust routes between nodes with demands on transportation services. Robust Shortest Path Problem (RSPP) specifications:

\[ \text{ADV Admissible Decision Variables: } x_{ij} \in \{0,1\} \text{ binary variables which assigns the arc } (i, j) \text{ to the shortest path; } z_{ij} \in [0,1] \text{ uncertainty level of } (i,j) \text{ link.} \]

Flow conservation principles for demands and admissible routing flows:

\[ \sum_{j} x_{ij} - \sum_{j} x_{ji} = \begin{cases} 1 & i \in [O] \setminus i \in [D] \setminus 0 & i \in O \cup D \end{cases} \]

Uncertainty Level:

\[ \text{W}_{T} = \{z_{ij} \mid ||z_{ij}||_{\infty} \leq \Gamma \}, \Gamma \in [0, \#T] \text{ is conservative level of the solution (i.e. the number of links influenced by travel times deviations)[17]. For } \Gamma = 0 / \Gamma = \#T \text{ we have respectively the strictly deterministic /fully deviated options [19-20].} \]

Performance criteria specifications: \[ Q = \sum_{ij} T_{ij}^{av} x_{ij} + \max_{z_{ij} \in \text{WT}} T_{ij}^{dev} x_{ij} z_{ij} \]

The RSPP optimization problem may be formulated as follows:

\[ \text{POmin } X, Z \text{ Q } (ADV)(FC)(ULC) \]
Operational Specifications: travel times representation, service time windows functionalities, customers visiting times sequence, maximum admissible routes distances/travel times are:

\[ t_{ik}^d \in [t_{si}, t_{ei}]; \quad t_{ij}^d + t_{dj}^d + \max[t_{ai} + \tau_i + T_{ij} - t_{si}, 0]x_{ij} \leq t_{ei} - t_{si} \]

OS):
The identical fleet of vehicles \( k=1,...,K \) (#K=m) specifications \( \forall k \in K: \) [limited capacity \( c_{pk} \) =cap]. The CVRP-TW optimization problem may be formulated as follows:

\[
Q = \sum_{i,j} \sum_{k} T_{ij} \cdot x_{ij}
\]

Remark: In this example the HILS upper layers interactions can offer the context related updated demand and network specifications (i.e. adequate level of service and routing parameters values for routing optimisation layer supported by intelligent monitoring and supervision layer activity. At the bottom direct control layer the DISCON dispatching control method can be adopted to the real-time routing disturbances compensations with routing treated as reference schedule to be stabilized by dispatching control actions. DISCON dispatching control: Reference scheduling specifications concerns the dynamic propagation off-schedule arrival \( t_{ij}^a \) /travel times \( T_{ij} \) deviations \( x_{ij} = t_{ij}^a - t_{ij} / z_{ij} = T_{ij} - T_{ij}^a + w_{ij} \) represented by punctuality control model and admissible deviations and control actions

(4)

At this point all DISCON dynamic dispatching control options (deterministic, stochastic, single/multi-criteria, robust, anticipative, priority control) are available (see [2-3][10-11][15]) for application. For example LQ/LOG control problems may be formulated as follows:

\[
PO \ min \ u_j \ J_{T-j} = \|x_T\|_{\Delta_T^2} + \sum_{k<j}^{\tau} x_k \|_{\Delta_k^2} + \|u\|_{R_k^2} \]

where \( Q_k, R_k \) are symmetric nonnegative definite weighting matrices, the first and the second term may be regarded as the off-reference scheduling deviations penalties at terminal (T) and all customers points. The last term penalize the weighted sum of squares of control actions.

3 ILLUSTRATIVE EXAMPLE

The presented model RCVRRTW was implemented to optimal routing problem in real network of Krakow city with real network traffic volumes. Due to lack of traffic real data concerning the dynamic traffic variability enabling the estimation of variability of the travel times these types of data was generated by traffic simulator Aimsun 8.0. This tool enables to realize the traffic micro-simulation. Fig. 4 presents the analyzed real network and graph model of this network. The traffic network inflow volumes values corresponds to measurements at the normal day and at morning rush hour from 2012 year realized by Infrastructure and Transport Authority in Cracow. Due to availability of these measurements and capabilities of Aimsun simulator it is possible to represent the travel times variability on network segments. In this network there are heterogeneous network segments from the point of view travel times variability e.g. measured by variances. In consequence the level of travel time uncertainty will be differentiated. For 11 randomly selected network nodes the robust routes was recognized by implementation of the model RSP. Based on these routes the optimal customer's allocations to vehicles servicing these routes are realized. The time windows were randomly selected from the interval [7, 14] with maximal admissible values of 1.5 (h). Similarly the service times was selected from [5, 10] minutes interval but demands from [50,350]. The vehicle capacity was equal 500 units. The optimisation was realised through Branch and Bound methods on the solver CPLEX 12.5. For illustrative purposes the solutions for two cases: classical deterministic and robust stochastic with uncertainty was compared. The results are presented in Tab.1. The travel times uncertainty consequences concerns not only changes in assignment of the given customer to other vehicle but the
changes of the sequence of the visited customers and generation of additional costs and decrease the reliability of service level.

Fig. 4. The analyzed area and its model - centre of Cracow.

Because the traffic processes are dynamic with existence of unpredictable traffic events (e.g. traffic incidents, blockings) the above solutions were verified randomly through Monte-Carlo method. For 1000 randomly selected values of variable $z_{ij}$ from [0,1] interval it was estimated the cost differences between classical deterministic and robust stochastic solutions. The results are presented in Fig. 5, Tab.1 and Tab.2 where the travel times reduction and number of positive/negative cases in Monte-Carlo method application are presented. As can be noticed the average benefit is equal to 4.16%, but maximal benefits is above 14%. The risk that the robust solution will be worse with respect to deterministic classical solution appears only 167 times on 1000, the maximal loss was 6.94%. The analysis confirms the sensibility of robust approaches to the vehicle routing problems.

Tab. 1. Classical and Robust: routing solutions

<table>
<thead>
<tr>
<th>Client sequences</th>
<th>Deterministic</th>
<th>Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route 1</td>
<td>1-161-9-90-1</td>
<td>1-9-64-1</td>
</tr>
<tr>
<td>Route 2</td>
<td>1-240-28-128-1</td>
<td>1-240-28-56-1</td>
</tr>
<tr>
<td>Route 3</td>
<td>1-182-56-200-1</td>
<td>1-128-50-200-1</td>
</tr>
<tr>
<td>Route 4</td>
<td>1-64-50-1</td>
<td>1-182-161-90-1</td>
</tr>
</tbody>
</table>
Fig. 5. Deterministic solution (left figure) and robust solution (right figure) for VRP-TW.

Tab. 2. Positive and negative aspects

<table>
<thead>
<tr>
<th></th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>-6.9337%</td>
</tr>
<tr>
<td>Mean</td>
<td>4.1635%</td>
</tr>
<tr>
<td>Max</td>
<td>14.0049%</td>
</tr>
<tr>
<td>No. Positives</td>
<td>833</td>
</tr>
<tr>
<td>No. Negatives</td>
<td>167</td>
</tr>
</tbody>
</table>

CONCLUSIONS

The logistic systems as a complex socio-technical system creates many high challenging tasks that properly solved can offer very high multi-criteria logistics activities related benefits. In this paper the proposal of the hierarchical multi-layer HILS platform is explored to solve the most frequently solved by different dedicated classic and metaheuristics tools vehicle routing problem. At present it is evidently visible lack of system-wide HILS functionalities related approaches which offer more representative solutions e.g. practically evident robust solutions of these problems. The reasonable computational time conditioned by the scale, routing specifications robust quality, efficiency of implemented solutions tools, lack of knowledge about system-wide interactions only theoretically enables the adequate passive reactions (e.g. re-routing actions) on unpredictable operational events (e.g. traffic incidents). However, the original proposal of dispatching control by DISCON method presented in this paper essentially improves of the above reactions by intelligent, real-time, anticipative, compensative context-related dispatching control actions offering very high practical benefits. In this context the presented by real example from Krakow network robust related benefits margins creates the premises for the professional selection of the control problems specifications.

Abstract

In this paper the proposal of implementation of the hierarchical multi-layers HILS platform dedicated to advanced ILS systems is presented. An illustrative example of the HILS platform multi-layers decomposition based robust routing problem VRP-TW was selected. In this case the upper HILS layers offer context-related network specifications that are included in the vehicle routing parameters and operational specifications updating used at optimization layer. The important advantages offered by HILS are problem consistency, lack of redundancy, reduction of problem dimension and inter-layers co-ordination. At the bottom dispatching
control layer the well known DISCON (Dispatching CONtrol) method from public transport was adopted to logistics applications with actual routing treated as obligatory reference schedule to be stabilized. The intelligence aspects are related among others to context-related trade-offs between routing modifications and corrective dispatching control capabilities e.g. priority or route guidance actions. The robust real example was presented and the robust features benefits are illustrated.

**VRP-R Problemy wyboru tras typu robust rozwiązywane na platformie HILS**

**Streszczenie**

W artykule propozycja zastosowania hierarchicznej wielo-warstwowej HILS platformy dedykowanej dla ILS systemów jest prezentowana. Ilustrujący przykład problemu wyznaczania tras w oknach czasowych VRP-TW typu robust wykorzystujący oferowaną przez HILS dekompozycję jest prezentowany. Górną warstwy HILS oferują specyfikacje sieciowe, które są wykorzystywane w warstwie optymalizacji dla estymacji potrzebnych parametrów i specyfikacji operacyjnych dla rozwiązywanego problemu optymalizacji tras. Istotne zalety proponowanego podejścia to kompatybilność, brak redundancji i między-warstwowa koordynacja. W dolnej warstwie sterowania dyspozytorskiego została zaadaptowana metoda DISCON z komunikacji zbiorowej dla powyższego problemu, traktująca propozycję tras z warstwy optymalizacji, jako "wirtualny rozkład jazdy, "który należy stabilizować przez działania sterujące np. wielokryterialne sterowanie priorytetowe na sygnalizowanych skrzyżowaniach ruchu. Obeznany rzeczywisty przykład z sieci Krakowskiej miał odpowiedzieć na pytanie ile możemy oczekiwać korzyśc od indywidualnych rozwiązań typu robust problemów wyboru tras. Uzyskane wyniki potwierdzają duży potencjał w rozwiązywaniu takich problemów.

**REFERENCES**


