Robust control of the inventory level in warehouses systems

INTRODUCTION

The modern ILS systems platforms hierarchical multi-layers structures offers many professional functionalities which directly to be a results of several important premises. This platform is embedded in an advanced sensing, information, computer, communication enabling technologies supported by capabilities of vehicle platforms (e.g. vehicle navigation, location, v-v. v-i communication, vehicle-probe etc.). Additionally it is supported by professional exploration of integration (co-operative systems actions with multi: networks/users/modes/services/objectives specifications) and intelligence (recognizing, diagnosing, understanding complex interactions and behavioral patterns). The proposals of development of multi-layer hierarchical HILS platform functional, physical and information structures was presented in ([4-7][10-16]). In this platform the fundamental obligatory general system principles: TRIAD I: Information-Integration-Intelligence; TRIAD E: Economy-Efficiency-Environment; TRIAD U: User-oriented-Up-to-date-Unified and dedicated ILS systems functionalities exploration inter-layers interactions are crucial for solution of challenging problems. The ILS system-wide inventory related option can be characterized by the following system layers specifications. Management and Coordination Layer: offers the Inventory Management System (IMS) functionalities through the provision of adequate strategies (network design, long-term implications, location of storages, obligatory standards), inventory policies focusing on the customers, consolidation/order picking/batching cost minimization policies. Adaptation Layer: The main premises of this layer is to guaranty the managerial and operational flexibility by decreasing the impacts of uncertainty (e.g. estimation and prediction of model parameters providing adequate representation for adequate evaluation, assessing and system-wide actions). In particular, estimation of representative demands patterns and customers satisfaction levels. Optimisation Layer: offer 2-D (time-space) inventory scheduling/ operational specifications to meet demands (compression, consolidation, storage capacity, online resources utilization). After updating the optimisation specifications (preferences, constraints, parameters robust values) the corresponding criteria and performance measures are selected for multi-criteria optimisation tools and implemented for solution of these problems. Intelligent Surveillance and Monitoring Layer: In this layer the real-time monitoring of the system environment is realized and multi-media technologies are used for visualization, warning and alarm generation purposes. Intelligent surveillance diagnoses abnormal situations and support the ILS system remedial actions. In consequence wide spectrum of professional anticipative and preventive actions practically on all system layers can be realized. Direct Control Layer: offer the dynamic control of inventory from several perspectives e.g. on-line exploration of the available inventory resources or demand prediction related real-time anticipatory ([1-3][8-9][13]) inventory level control activity presented in this paper.

1 INVENTORY COSTS FORECASTING BY ARIMA MODELS

From several types of logistics costs the storage costs have significant importance on the overall financial condition of enterprises. Irrational inventory management policies can contribute to a significant increase of inventory costs. Basically we can distinguish costs associated with inventory creation, maintenance and shortage. These groups also contain the cost incurred as a result of delayed delivery, the penalties for lack of products, the lost sales opportunities cost and costs of product
obsolescence. Very often, the level of inventory costs incurred in the enterprise are determined by the impact of external factors (prices of external logistics services including warehousing, transportation, services, interest rates on loans to finance inventories) [19]. In practice, it is not considering inventory costs separately with division into their individual components. Therefore, keeping accurate forecasts of inventory costs allows to continuously improvement of logistics processes and making more effective decisions in this field. For solution of practical problem the data collection and analysis play a crucial role. Because of data random nature most of the surrounding us business processes are considered as a stochastic processes. Stochastic process is a random time function \( X(t, \omega) \) where: \( t \in T \) and elementary events \( \omega \in \Omega \) belongs to probability space \( (\Omega, F, P) \). It is said that the time series used to build forecasts are realizations of the discrete type of stochastic processes. The time series shows the course of the process in one of the many possible cases. ARIMA Models (Auto - Regressive Integrated Moving Average) are advanced forecasting tools, suitable for modeling processes characterized by high dynamics and the difficulty in identifying relations which are shaping course of the process. ARIMA models are based on autocorrelation phenomenon i.e. the correlation of variable values forecast with the values of the same variable delayed in time. The main characteristic feature of ARIMA models is the fact that the value of the predicted variable at time \( t \) is a linear combination of values of the same variable from previous periods \( T = \{t-1, t-2, \ldots, t-p\} \) increased by a certain value of the random component. Among such models we can distinguish three basic types: autoregressive models (AR): moving average models (MA): mixed above autoregressive and moving average models (ARMA). In general the auto-regressive model of order \( p \) can be represented as:
\[
Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t
\]
where: \( Y_t, Y_{t-1}, Y_{t-2}, Y_{t-p} \) - the value of the variable, respectively, at the time \( t \in T_p; \{\phi_i\}_{i=1}^{p} \) - model parameters; \( e_t \) - is a dynamic random component but \( p \)- order lag. Delay parameter \( p \) determines how far back in time should we reach to determine the value of the forecasted variable at time \( t \). If the random components of the past are correlated we have to deal with the process of moving average (MA), which is expressed by the formula:
\[
Y_t = \theta_0 - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} + e_t
\]
where: \( e_t, e_{t-1}, e_{t-2}, e_{t-q} \) - residuals of the model respectively at the time period \( t \in T_q; \{\theta_i\}_{i=1}^{q} \) - model parameters, \( q \) - order lag. In order to better adaptation of the model to the historical data the connection of (AR) and (MA) parts of the model is often made. It is known as the ARMA model having both a \( p \) and \( q \) parameters. This combination may be represented as follows:
\[
Y_t = \varphi_0 + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + e_t - \theta_0 - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}
\]
In this model, the predicted values of the variables in time period \( t \) depends on the values of the past and residues of the model in the previous time instants. To apply ARIMA models the stationary condition of the examined time series representing economic process is required. By stationary, we can understand the constant time variance and the expected value of the process. To investigate the station-ary of inventory costs data from energy company the extended statistical Dickey Fuller test has been applied. It is known as a unit root test, taking into account possible autocorrelation of the random component. The null hypothesis (H0) assumes lack of stationary of examined variable and the alternative hypothesis (H1) indicates that the variable is stationary. Null hypothesis is rejected if the calculated value of the ADF test statistic is less than the critical value read from the appropriate tables for the adopted level of significance. Detailed description of test can be found in [17-18]. When the stationary condition is not satisfied the examined time series should be reduced to stationary by calculating the following differences: \( \Delta y_t = y_t - y_{t-1} \) the first differences, \( \Delta^2 y_t = \Delta y_t - \Delta y_{t-1} \) the second differences. If the calculation of first differences is sufficient to achieve stationary states it is said that the series are integrated of order one (1). Fig. 1 presents a summary of the monthly inventory cost in the company of the energy sector in the years 2004-2010.
Fig. 1. Inventory costs between 2004 and 2010

For the above data the extended stationary Dickey Fuller test was carried out to determine the degree of integration of the series. The inference was made for the significance level $\alpha = 0.05$. In this case, the p-value (talking about the level of significance for which the null hypothesis can be rejected) at a level less than 0.05 indicates stationary of the series. In this example, the p value was 0.63 so that the series are not stationary. Therefore, it should be reduced to stationary by the operation of differentiation. In addition, in order to stabilize the variance the values of time series have been squared. The specified in this way time series is shown in Fig. 2. Thus obtained series was again subjected to a statistical unit root test. Obtained p-value equal to $4.33 \cdot 10^{-22}$ is less than 0.05. Therefore, after applying the operation of differentiation the considered time series was reduced to stationary and it is integrated of order one I(1). The initial step of forecasting by the use of autoregressive methods is the identification of model parameters of the ARIMA model. For this the analysis of the autocorrelation function (ACF) and partial autocorrelation function (PACF) of time series is made and based on the methodology of Box Jenkins (Fig.3). According to this methodology, the number of statistically significant parameters of the function PACF suggests the number of delays (parameter q) in part of MA-ARIMA model. The number of statistically significant parameters of the function ACF suggests to choose the number of delays (parameter p) in part of AR-ARIMA model. For the autocorrelation function significantly different from zero coefficient is for lag number one, remaining are not significantly different from zero, so you can skip them. In the case of partial autocorrelation function significantly different from zero coefficient are for delays 1 and 2. At the border of significance is a parameter for delay 4. On the basis of previously conducted reasoning can be assumed that the most appropriate model will be ARIMA (1,1,4). Therefore, the selected prediction model of inventory costs for that company can be summarized as follows:

$$Y_t = \varphi_1 Y_{t-1} + \varepsilon_t - \theta_0 - \vartheta_1 \varepsilon_{t-1} - \vartheta_2 \varepsilon_{t-2} - \vartheta_3 \varepsilon_{t-3} - \vartheta_4 \varepsilon_{t-4}$$ (4)

Estimates of the parameters was carried out by maximum likelihood, which allows to verify the adopted model. The maximum likelihood estimator is the value of the parameter that maximizes the likelihood function; it is often easier to work with the log-likelihood, and this causes no problems because the value that maximizes the likelihood function also minimizes the log-likelihood [18]. To estimate the parameters of the ARIMA (1,1,4) model the data from 79 values of the variable have been used, while the positions from 80 to 82 were used to verify proposed method. Finally, the prediction model can be formulated as follows:
Fig. 2. The time series after squared and differentiation operation

Fig. 3. Autocorrelation (ACF) and Partial autocorrelation functions (PACF)

Fig. 4. The original and predicted values of the inventory costs in the considered company
The obtained model can be used for predictions. The original adjusted data are shown in Fig. 4. In order to verify the correctness of ARIMA model parameters selections the autocorrelation analysis of the model residues were conducted. The lack of significantly different from zero auto-correlation coefficients for subsequent lags indicate that the model parameters were correctly selected. Fig. 5.

In order to evaluate the fit of the model to empirical data the determination coefficient $R^2$ has been calculated. Its value is $R^2=0.75$ On this basis it can be concluded that the estimated model is relatively well matched to the actual data.

![Model Residuals ACF](image)

![Model Residuals PACF](image)

Fig. 5. Autocorrelation (ACF) and (PACF) for residuals of the model

2 IMPLEMENTATION OF THE SPECTRAL ANALYSIS TECHNIQUES FOR PROGNOSIS

The above presented popular prognosis time series based methods gives positive results only in specific cases. In the observed worth globalization and fast dynamic changes at the markets many economic processes have complex specifications (fast dynamic, randomness, high level of uncertainty complex interactions etc.). The logistics delivery networks management (demand prognosis, costs, procurement logistics, stock control etc) is in this field very good example. To guarantee of the required level of system efficiency and customers level of service the exploration of the representative information for estimation and prediction of the adequate processes parameters is a crucial premise to achievement the necessary logistics activities efficiency and productivity. Time series have potential of describing the dynamic phenomena by combination of fixed, cyclic, random and trend components however the interference of several cyclic components with different frequencies and random components frequently gives not accurate prognosis. The basic area of the spectral analysis is the cyclical nature of the processes. Assuming the wave structure of the stochastic processes it is possible to analyze time series in the frequency domain. The application of the harmonic $n/2$ for $n$ observations (first with period $n$, second $n/2$ etc) is based on the assumption that time series is stationary. If this necessary condition is not satisfied the trend removing or differential operation can be used representing the process in the following way [20]:

$$y_t = f(t) + \sum_{i=1}^{n/2} \left[ a_i \sin \left( \frac{2\pi i t}{n} \right) + b_i \cos \left( \frac{2\pi i t}{n} \right) \right]$$

(1)
where: $i$ - number of harmonic; $a_1$, $b_1$, $a_2$, $b_2$, … - fixed values, $f(t)$ – the tendency function

The parameters values $a_1$, $b_1$, $a_2$, $b_2$ are receiving by least square method from formulas:

$$a_i = \frac{2}{n} \sum_{i=1}^{n} y_t \sin \left(\frac{2\pi}{n} it\right) \quad dla \ i = 1 \ldots \frac{n}{2} - 1$$  \hspace{1cm} (2)

$$b_i = \frac{2}{n} \sum_{i=1}^{n} y_t \cos \left(\frac{2\pi}{n} it\right) \quad dla \ i = 1 \ldots \frac{n}{2} - 1$$  \hspace{1cm} (3)

$$b_{n/2} = \frac{1}{n} \sum_{i=1}^{n} [y_t \cos(\pi t)]$$  \hspace{1cm} (5)

For last harmonic we assume the values: $a_{n/2} = 0$  \hspace{1cm} (4). Basing on the discrete time series Fourier transformation we receive the frequency spectrum of the given process which indicate the dominating frequencies for analyzed series variability (e.g. variance)[21]. The portions of the variance generating by $i$-th harmonic is calculated by formulas:

$$\omega_i = \frac{a_i^2 + b_i^2}{2\sigma^2} \quad dla \ i = 1 \ldots \frac{n}{2} - 1$$  \hspace{1cm} (6)

$$\omega_i = \frac{a_i^2 + b_i^2}{\sigma^2} \quad dla \ i = \frac{n}{2}$$  \hspace{1cm} (7)

where: $\sigma^2$ - variance of forecasting variable after trend subtracted

![Fig. 6. The demand profile for product during two months](image)

![Fig. 7. The spectrum of the analyzed time series](image)

**Example:** The efficiency of the proposed spectral analysis approach was verified on the real data from enterprise. The Fig. 6 presents the daily profile of the given product sale. From the analysis of
this graph we can come to the conclusion that supply pattern on this product is probably generated by many different cycles and random fluctuations. To recognize the fundamental cycles influences the spectrum of the analyzed series is determined after the trend removing. This spectrum was smoothing by Bartlett window with zero values of the terminal elements, to recognize the frequency areas of the maximal spectral frequencies (Fig.7). From this figure we can to select the three areas in which the harmonic have the maximal spectral density, and as a result have maximal impact on the series form. For prognostic model creation only selected harmonics was used (see Tab.1)

Tab. 1. Harmonics implemented to prognostic model creation

<table>
<thead>
<tr>
<th>No of selected harmonic</th>
<th>Denoting</th>
<th>Cycle length [days]</th>
<th>No of selected harmonic</th>
<th>Denoting</th>
<th>Cycle length [days]</th>
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<tbody>
<tr>
<td>1</td>
<td>n</td>
<td>60</td>
<td>17</td>
<td>n/17</td>
<td>3,53</td>
</tr>
<tr>
<td>2</td>
<td>n/2</td>
<td>30</td>
<td>19</td>
<td>n/19</td>
<td>3,16</td>
</tr>
<tr>
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<td>20</td>
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<td>3</td>
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<td>21</td>
<td>n/21</td>
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<tr>
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<td>22</td>
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<td>n/25</td>
<td>2.4</td>
</tr>
<tr>
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<td>n/14</td>
<td>4.29</td>
<td>26</td>
<td>n/26</td>
<td>2.31</td>
</tr>
<tr>
<td>15</td>
<td>n/15</td>
<td>4</td>
<td>28</td>
<td>n/28</td>
<td>2.14</td>
</tr>
</tbody>
</table>

After parameters estimation the following prognostic model is received

\[ y_t = 0.0722t + 22,516 - 3.9 \sin\left(\frac{2\pi}{60} t\right) + 2.5 \cos\left(\frac{2\pi}{60} t\right) + 3.39 \sin\left(\frac{2\pi}{2t} t\right) - 3.36 \cos\left(\frac{2\pi}{2t} t\right) + 3.65 \sin\left(\frac{2\pi}{7t} t\right) + 4.55 \cos\left(\frac{2\pi}{7t} t\right) - 0.51 \sin\left(\frac{2\pi}{8t} t\right) + 5.48 \cos\left(\frac{2\pi}{8t} t\right) + 4.33 \sin\left(\frac{2\pi}{9t} t\right) - 5.26 \cos\left(\frac{2\pi}{60t} t\right) + 5.78 \]

\[ \sin\left(\frac{2\pi}{60} 12t\right) - 0.54 \cos\left(\frac{2\pi}{12t} 12t\right) - 2.81 \sin\left(\frac{2\pi}{60} 13t\right) + 1.68 \cos\left(\frac{2\pi}{60} 13t\right) - 4.69 \sin\left(\frac{2\pi}{60} 14t\right) - 2.28 \cos\left(\frac{2\pi}{14t} 14t\right) + 5.14 \sin\left(\frac{2\pi}{60} 15t\right) + 1.09 \cos\left(\frac{2\pi}{60} 17t\right) + 5.45 \sin\left(\frac{2\pi}{60} 17t\right) - 0.42 \cos\left(\frac{2\pi}{60} 17t\right) - 0.94 \sin\left(\frac{2\pi}{60} 19t\right) - 4.7 \cos\left(\frac{2\pi}{60} 20t\right) - 3.08 \sin\left(\frac{2\pi}{60} 20t\right) - 1.74 \cos\left(\frac{2\pi}{60} 21t\right) - 3.08 \sin\left(\frac{2\pi}{60} 21t\right) - 1.14 \cos\left(\frac{2\pi}{60} 22t\right) + 0.38 \sin\left(\frac{2\pi}{60} 22t\right) - 4.29 \cos\left(\frac{2\pi}{60} 22t\right) + 5.22 \sin\left(\frac{2\pi}{60} 23t\right) - 1.66 \cos\left(\frac{2\pi}{60} 23t\right) + 3.02 \sin\left(\frac{2\pi}{60} 25t\right) + 2.48 \cos\left(\frac{2\pi}{60} 25t\right) - 1.38 \sin\left(\frac{2\pi}{60} 26t\right) - 6.19 \cos\left(\frac{2\pi}{60} 26t\right) + 3.45 \sin\left(\frac{2\pi}{60} 28t\right) - 0.52 \cos\left(\frac{2\pi}{60} 28t\right) \]
Fig. 8. Theoretical model values with prognosis

Tab. 2 Comparison of the prognosis errors for the selected models

<table>
<thead>
<tr>
<th>Average prognosis error</th>
<th>Prognosis Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model Browns*</td>
</tr>
<tr>
<td>Absolute</td>
<td>17</td>
</tr>
<tr>
<td>Relative</td>
<td>1,36</td>
</tr>
</tbody>
</table>

* smoothing constant - 0.9

The above model can be implemented for prognosis future demands on the selected product. The theoretical values from model together with prognosis are presented in Fig. 8. From this figure it is evident that prognosis to high degree is compatible with the real time series. For analyzed example the other prognostic models were developed with accuracy presented in Tab. 2.

3 ILS SYSTEMS RELATED OPTIMAL CONTROL OF THE INVENTORY LEVEL

The inventory operational environment dynamic control problems are related to: throughput fluctuating demand pattern (e.g. relocation of pallets with high expectancy of retrieval in a future to be closer to the pickup and delivery points), re-scheduling of the initial random storage assignment by shifting nearer pickup and delivery points more frequently accessed products, dynamic reconfiguration of the order picking system. The typical requirements addressed to control actions concerns: flexibility (due to reconfigurations, reassignments), real-time control of the system behavior (order picking, location of goods) and scheduling demand specified related real-time dedicated control [11-12]. In this paper basing on the above presented prognosis of demand the robust control of the inventory level minimizing the ordering and holding costs is formulated and solved. From the Inventory Management and Coordination layer the Suppliers (S)–Inventory/Vendor (I)–Customers (C) chain specifications are estimated e.g. for each Si supplier lead time/ordering costs. The intuitive problem may be stated as how much and when to order and which suppliers to select to realize of this order to minimize the total costs. The minimized expenses of the vendor are the costs of holding and inventory and the ordering costs. The income of the vendor is equal to difference of the sales profit (e.g. product of price and demand SP(t)=p^s d(t)) I(d(t)-I(t)) with I(.) to be Kronecker delta minus the holding costs (e.g. product of price per unit inventory and size I(t) of available inventory HC= p^s I(t) ) and ordering costs (e.g. fixed part of replenishment order plus order size and time delay dependent for delivery of that order by selected supplier i=1,...,N_S: OC=\sum_{t=L_i}^{t-L_i} \sum_{i} \{c_{i}^o + p_{i}^o u_{i}^o (t-L_i)\} where the u_{i}^o (t-L_i) is quantity of the goods ordered from supplies S_i at time t with delivery time L_i. In this paper more complicated dynamic discrete time control problem is formulated and solved: Problem Assumptions: (A1): Inventory level in time period k evolution I(k)≥0 with initial level I(0)=0 and rises when the supplier order arrives/falls with d(k) when a sale is made. (A2): The lead time L_i function for the delivery time for an order placed at S_i typically follows a certain probability distribution, (A3): d(k) demand function has two stochastic sources i.e. demand quantity and frequency to be usually represented by normally distributed probabilistic representations. In the ILS system adaptive layer the customer satisfaction level estimation and prediction specifications transferred to direct control layer
influences the above probabilistic time-dependent normal white noise demand robust parameters values.

**Problem specifications:** Single Stocking Point: Single-Commodity; No backlogging is allowed. The control inventory state (CIS) model has the form $x_{k+1} = x_k + u_k - L + z_k - d_{k+1}$ (CIS), where state variables represent the inventory-demand surplus deviations $x_k = I_k - d_k$ and $z_k - d_{k+1}$ white noise type stochastic processes related to predicted demands. The state and control variables are bounded by constraints: $x_k \in [x_L, x_U], u_k \in [U, V]$ (SC). Performance criterion: the sum of inventory surplus related holding costs and demand surplus related ordering costs including the costs of delivery time (i.e. admissible supplier with minimum lead time will be selected). The following LQG type control problem is formulated: where $Q_k$, $R_k > 0$ and $S_k > 0$ are weighting matrices.

$$\text{PO}_{\min} u \ J = \sum_{k=0}^{\infty} \|x_k\|_{Q_k}^2 + \|u_k\|_{R_k}^2 + \|u_k - u_{k-1}\|_{S_k}^2 \quad \text{(CIS) (SC)}$$ (1)

The first and second terms penalize both the weighted sum of squares of inventory and demand surplus actions and the weighted sum of squares of the first backward differences in control actions (i.e. the costs of the magnitude of control actions and adaptive control smoother features). This control actions are embedded in the bottom direct control layer of the ILS system (see Fig. 9).

The control results for the selected demand prognosis presented above are illustrated in Tab.3

Tab.3 The results of solution LQG inventory control problem
CONCLUSION

The stochastic inventory level control problem supported with references created by representative demand prediction is a good practical approach to basic cost-effective inventory operation problem. The proposed problem formulation includes main operational cost generating premises and can be extended by customers satisfaction components influencing the future demands levels.

Abstract

In the paper the inventory level optimal control problem was presented and solved. The crucial control specification is created by representative prediction of stochastic demand of the goods buyers which creates the reference for adequate control problem formulation. The minimized control performance criterion represents the sum of inventory surplus related holding costs and demand surplus related ordering costs including of delivery time, which directly leads to optimal inventory level optimization. For this problem the stochastic demand prediction problems were solved and verified by adequate models. After that the dedicated dynamic stochastic inventory level control problem with predicted demand treated as references was solved. The original results confirm high practical utility of the proposed approach.

REFERENCES