INTRODUCTION

In gears the backlash in mesh may cause a loss of the contact between the meshing teeth and generation of vibration (so-called rattle).

The generation of the vibration in gears is due to separation of teeth on the theoretically correct line of action. In case of instantaneous tooth engagements (so called vibro-impacts) the vibration can appear on the second line of action, which corresponds to the reversible motion in relation to the nominal rotational motion of the gear. This phenomenon increases levels of vibration and noise, especially in unloaded and lightly loaded gears e.g. in vehicles and machine tools etc. An investigation into the nature of these vibrations has been a subject of many research works (e.g. [1÷4]), and is important for the means of the transport in which the gears are applied, so, broadly speaking, for the transport.

There are i.e. parameters which are conductive to loss of contact between the meshing teeth:

– a kind of the vibration forcing and loadings (variable, light),
– rotational velocity adequate for operation at the resonant zones.

There are also a number of examples of gears for which may occur the vibration problems due to possible loss of contact between the meshing teeth. These are among others gearboxes in vehicles in case of idle running (so-called neutral gear rattle), gears with significant base pitch deviations and the gears operated at resonant zones by the constant external loads. This last case is now our main focus of interest.

1 DISTANCE LAG ON THE SECOND LINE OF ACTION FOR THE GEAR MESHING STIFFNESS

It is reasonable to demonstrate the way of modelling the gear mesh by the use of two systems of springs, which are mutually shifted. Now, let’s imagine meshing teeth when rattle vibrations occur. Let the driving flank of the pinion take position exactly at the pitch point C. In this position a tooth of the wheel being in engagement with a tooth of the pinion, overcomes, due to excitation forces, the distance of the backlash that results in the instantaneous contact between flanks of teeth which are opposite to the driving ones. So, it is the instantaneous contact of the teeth of the wheel with the tooth of the pinion going before.

Analysing the trajectory of the tooth of the wheel we can determine its position as a point of intersection of the tooth profile (‘stopped’ in thought at the moment of an impact) with the second line of action that determines reversible motion of the gear. When we know the co-ordinates of this point, we can locate the position of the tooth that follows the one being under investigation. Thereby we can determine the phase of the mesh on the first and second line of action, as well as find the phase lag in the second stiffness function in relation to the first one.
The complicated description of this problem can be significantly simplified, if we introduce a system of co-ordinates through the axis of symmetry of the considered tooth of the wheel. This makes it possible to calculate a new action angle $\alpha_\Sigma$. This angle, in relation to the turned and displaced system of co-ordinates, is a sum of the pressure angle $\alpha_w$ and the angles corresponding to the backlash as well as a half of thickness of a tooth on the rolling radius. We can also notice, that the problem showed in this way, does not require considering the geometry of the pinion, but only the tooth profile of the wheel and the second theoretically possible line of action for the reversible motion. The system of co-ordinates is shown in the Fig.1. It has already been displaced and rotated, which gives the most convenient position for next calculation (Fig.2).

Fig.1. Teeth of the pinion and wheel at the pitch point $C$ in the both possible lines of action (left side of the picture - motion in the right direction ) – an original coordinate system, *source: The Author’s material*

Fig.2. The actual position of teeth of the wheel on the second line of action after rotation of the original coordinate system, *source: The Author’s material*
In the analytical description of an involute tooth profile one may use the formulae defining thickness of the tooth on an arbitrary chosen radius, with the assumption that the teeth engage one with another on the involute shape flanks. A range of calculation is limited by the value of addendum radius \( r_{a2} \) as the upper bound and the rolling radius \( r_{w2} \) as the lower one. The calculations are finished, when the point \( R \) that belongs to the tooth profile and simultaneously to the second line of action is achieved.

Thus, in the calculation we should take into consideration:

- the tooth thickness of the wheel on the pitch diameter \( (m_n \) - module pitch of a gear, \( x_2 \) - tool displacement factor for toothed wheel, \( \alpha \) - normal profile angle)

\[
\hat{g}_{x2} = m_n (\pi / 2 + 2x_2 \tan \alpha)
\]  

(1)

- the angle corresponding to the half of tooth thickness on the considered radius \( r_r \)

\[
\varphi_r = \frac{\hat{g}_{x2}}{d_2} + \text{inv} \alpha - \text{inv} \alpha_r
\]  

(2)

where, \( \text{inv} \alpha = \tan \alpha - \alpha \) and \( \text{inv} \alpha_r = \tan \alpha_r - \alpha_r \)

- the profile angle on the radius \( r_r \)

\[
\alpha_r = \arccos \left( \frac{r_{b2}}{r_r} \right)
\]  

(3)

where, \( r_{b2} \) - a radius of the base circle

- an equation of the second line of action

\[
w = \tan \alpha_x v + r_{w2} [\cos(\Phi / 2) + \sin(\Phi / 2) \tan \alpha_x ]
\]  

(4)

where \( \Phi \) - an angle corresponding to the sum of the thickness of the tooth on the rolling radius and of the double backlash (Fig.2)

- \( \text{co-ordinates of intersection of the second line of action with the tooth profile on a certain radius } r_R \)

\[
w_R = \tan \alpha_x r_R \sin \varphi_R + r_{w2} - h + H
\]  

(5)

- \( \text{co-ordinates determined on the basis of an angle } \varphi_r \)

\[
w_r = r_r \cos \varphi_r
\]  

(6)
Fig. 3. The actual position of the teeth in mesh on the second line of action after rotation and translation of the original coordinate system, source: The Author’s material

If \( w_r - \Delta w \leq (w = w_R) \leq w_r + \Delta w \), when accuracy of the calculations reaches \( \Delta w = 10^{-6} \) up to \( 10^{-7} \) m, then the value of the distance which corresponds to the phase lag in a time history of varying meshing stiffness (subindex \( r \) instead of \( R \)) is determined by the formula

\[
|\mathcal{CR}| = \sqrt{(r_R \sin \varphi_R - r_{w2} \sin (\Phi / 2))^2 + (r_R \cos \varphi_R - (r_{w2} - h))^2}
\] (7)

The outlined method of description of the distance \(|\mathcal{CR}|\) (Fig. 3) which corresponds to the time lag in the time history of varying meshing stiffness and in this way the mesh on the second line of action is valid in cases when tooth flanks with involute profiles take part in engagement and the addendums of the teeth have not been significantly shortened and when the actual deflections of teeth are excluded.

2 FORMS OF THE EQUATIONS – AN ANALYSIS OF MOTION

The equation in case of loss of contact for the meshing teeth, for mesh backlash and random distribution of base pitch errors in each of the wheels, can be introduced in translation co-ordinates

\[
\frac{d^2 y}{dt^2} + \frac{c_g}{M_Z} \frac{dy}{dt} + \frac{k_{gl}}{M_Z} f(t, y, y_{er}, y_{\eta}) = \frac{P}{M_Z}
\] (8)

where: \( y \) - relative displacements between gears, \( M_Z \) - equivalent mass reduced on the input shaft; \( c_g \) - equivalent damping; \( k_{gl} \) - meshing stiffness of the single pair of teeth ([5],[6]); \( f(t, y, y_{er}, y_{\eta}) \) - function describing the mesh and a change of the global meshing stiffness dependent of time, displacements, deviations of teeth and of the backlash; \( P \) - constant meshing force as a result of the static balance of the input and output torques.

In case of spur gears the static deflection \( y_{st} \) is determined by the use of the stiffness of the single-pair teeth.

When displacements, the function of the base pitch errors and the mesh clearance are expressed in relation to the static deflection \( (y^* = y / y_{st}, \ y_{er^*} = y_{er} / y_{st}, \ y_{\eta^*} = y_{\eta} / y_{st}) \), for non-dimensional time \( t^* = \omega_n t \), meshing frequency \( v = \Omega_1 z_1 / \omega_n \), angular input velocity \( \Omega_1 \), numbers of teeth of the pinion and wheel \( z_1, z_2 \) and the period \( T^* = 2\pi / v \), it is possible to obtain the equation in the for
\[ \frac{d^2 y^*}{dt^2} + 2 \zeta \frac{dy^*}{dt} + k(t) f[y^*, y_{m1}, y_{m2}, y_{n}] = B_0 \]  

(9)

where, \(\zeta\) - dimensionless damping factor, \(\omega_n\) - frequency for the single-pair teeth stiffness, \(B_0\) - constant load.

Excluding deviations relevant to corresponding flanks of the gear teeth, the mesh function for constant stiffness may be described in the formula

\[
F(y, y_{\eta}) = \begin{cases} 
  y, & y > 0 \\
  0, & -y_{\eta} \leq y \leq 0 \\
  y + y_{\eta}, & y < -y_{\eta}
\end{cases}
\]  

(10)

### 3 NUMERICAL EXAMPLES

The examples have been chosen in order to show dynamic factors in case of acting constant external loads and the mesh stiffness assumed to be rectangular wave form. At the beginning of the calculation the model without the distance lag \(|C\pi|\) has been examined.

![Fig. 4. The dynamic factors: \(K_{dyn}\) – for maximal forces on the normal line of action and formally negative on the second one, source: The Author’s material](image)

In this case, by the low levels of the mesh damping (Fig. 4 - the dimensionless backlash \(\eta = 1.75\), \(\varepsilon = 1.5\) and different levels of the non-dimensional damping \(\zeta = 0.025 \div 0.05\)) even the rattle vibration take place at the rezonant zones. By the higher levels of damping, at the same bands of the non-dimensional frequency \(\nu\), we can observe only a loss of contact between the meshing teeth and so-called single–sided impacts.
In computation we have introduced the factors of loss of contact for the meshing teeth \( w_1 \) as the relation of the total time of loss of contact between the teeth to the total time of the numerical realization for each value of the mesh frequency (Fig. 5). It is visible that an instantaneous lack of contact takes place at the resonant zones (the same bands of the non-dimensional frequency \( \nu \) approximatelly equal to \( \frac{1}{4} \), \( \frac{1}{2} \), 1, 2…) by the low damping levels and even by the \( \zeta = 0.1 \) in the main resonant region of frequencies. Under the existing description of the distance lag on the second line of action (\( |\mathcal{R}| \neq 0 \)), by the same levels of damping and with the various clearances, only by the low damping and clearances (\( \eta_{\text{max}} \approx 1.75 \pm 2.4 \)) the rattle vibration can be excited and so-called double-sided impacts can be observed.

The example illustrated in the Fig.6, shows the gear with random base pitch deviations \( |f_{z1i}| < 0.5 \) and \( |f_{z2j}| < 0.6 \) (ranges of values of the base pitch deviations for both toothed wheels as a relation to the static deflection \( i = 1, ..., z_1; j = 1, ..., z_2 \)).

In this case for \( B_0 = 1 \), random pitch deviations for both wheels \( |f_{z1i}| \leq 0.5, |f_{z2j}| \leq 0.6, \zeta = 0.1, \eta = 1.75, \varepsilon = 1.5 \) and the smallest common multiple SCM \( (z_1, z_2) = 550 \) there have been computed:
- \( K_{\text{dmax}} \) – dynamic factors for maximum dynamic force in each numerical realisation,
- \( K_{\text{dav}} \) – average dynamic factors for all maximum forces in each mesh,
- \( K_{d1\sigma}, K_{d2\sigma}, K_{d3\sigma} \) – average for all values of dynamic forces enlarged by the multiple of the standard deviation,
- \( w_1 \) – the formal factor of loss of contact for the meshing teeth.
Fig.6. Dynamic factors: $K_{dmax}$ – maximum dynamic forces, $K_{dav}$ – average for all maximum forces in each mesh, average ones for all value of dynamic forces enlarged by the multiple of the standard deviation $- K_{d1\sigma}, K_{d2\sigma}, K_{d3\sigma}$ and the factor of loss of contact of the meshing teeth $w_1$, source: The Author’s material.

CONCLUDING REMARKS

In the light of research on the numerical models of gears (the gears used in the means of transport), the following conclusion can be drawn: the vibrations for which loss of contact of meshing teeth take place, can develop even by the constant external loads - mainly at the resonant zones by small backlash and for the low damped system. The parameters that affect generation of these kind of vibrations, are also the random pitch deviations of both toothed wheels. The most frequent is loss of contact on the driving side of the tooth and it results then in so-called single-sided impacts. For the suitable combination of external forces, backlash, varying meshing stiffness as well as for low damping, so-called double-sided impacts take place. In this case the opposite flank to the driving one (driven side of the tooth) can be in the instantaneous mesh.

Abstract

The forced model of an isolated single-stage gear considering backlash and possible instant engagements of teeth on the second line of action is examined. Base pitch errors, various levels of viscous damping and the description of the varying gear meshing stiffness are taken into consideration. Results of calculation for the suitable combination of external forces, backlash, varying meshing stiffness, base pitch errors as well as damping, show that the phenomenon of possible loss of contact between the meshing teeth can be generated at the resonant zones even by the constant external loads.

Utrata kontaktu pomiędzy zębami w modelu przekładni o zębach prostych

Streszczenie

W pracy zbadano model izolowanej przekładni zębatej z uwzględnieniem luzu międzyżebnego oraz możliwości chwilowego styku zębów na drugiej teoretycznej linii przyporu. Uwzględniono również opis błędów podziałki zasadniczej, różne poziomy tłumienia dźgań przekładni oraz zmienność w czasie sztywność międzyżebną.

Dla odpowiednich kombinacji sił zewnętrznych, luzu, zmiennej sztywności, błędów podziałki zasadniczej jak też tłumienia, zderzanie styku zębów występuje w strefach rezonansowych nawet przy wymuszeniu stałym.
obciążeniem zewnętrznym.

REFERENCES