INTRODUCTION

Digital signal processing methods are widely used in many branches of science and technology [5, 6]. The problem of image compression remains one of the most important problems of this field [2, 6, 7]. The need of decreasing the image size without a noticeable loss of its quality is still a challenge. Among the variety of compression methods the transform methods belong to the most effective ones [2, 7]. They represent the class of lossy methods, where the reconstructed image is always affected by the compression process. Dependent on the used compression method and the type of the transform, the compression effectiveness can vary in a wide range. The typical way to evaluate the compression quality is to compare the reconstructed image with the original one. Typical measures used for such a comparison, as: Mean Square Error (MSE), Normalised Mean Square Error (NMSE), Signal-to-Noise Ratio (SNR) and Peak Signal-to-Noise Ratio (PSNR) are objective evaluation tools [6]. Besides them, it is necessary to use a visual inspection in order to perform a subjective evaluation of the compression effectiveness.

1 DEFINITIONS OF THE SELECTED PIECEWISE-LINEAR TRANSFORMS

Piecewise-linear transforms, like every other transformation, can be presented as mapping of a signal onto the appropriate set of basis functions [1]. The transformations considered in the paper are based on sets of piecewise-linear functions, introduced by Dziech [1] in the eighties of the XX century. Two transformations are considered in the paper:

– the PWL (Periodic Walsh Piecewise-Linear) Transform,
– the HPL (Haar Piecewise-Linear) Transform.

1.1 The PWL Transform

The PWL transform represents mapping of the signal onto the set of PWL basis functions. They are obtained by integrating of the Walsh functions and supplementing the obtained set with the constant function equal to 1. In the continuous case, expansion of the signal $x(t)$ into the PWL series is expressed by the following formula [7]:

$$x(t) = \sum_{i=0}^{N-1} c_i \cdot PWL(i, t)$$

(1)

where:

$c_i$ - the PWL spectrum of the signal $x(t)$.

The spectral coefficients $c_i$ are defined by following relationships:

$$c_0 = \frac{1}{T} \int_0^T x(t)dt$$

(2)
\[ c_i = \frac{-1}{2^{k+i}} \int_0^T x(t) \cdot wal'(i,t) \, dt \] (3)

where:
- \( wal'(i,t) \) – derivatives of continuous Walsh functions;
- \( i \) – number of Walsh function, \( i = 1, 2, ..., N-1 \);
- \( k \) – group index of the PWL functions,
  \( k = 1, 2, ..., \log_2 N \).

1.2 The HPL Transform

The HPL transform represents mapping of the signal onto the set of HPL basis functions. They are obtained by integrating of the Haar functions and supplementing the obtained set with the constant function equal to 1. In the continuous case, expansion of the signal \( x(t) \) into the HPL series is expressed by the following formula [2]:

\[ x(t) = \sum_{i=0}^{N-1} c_i \cdot HPL(i,t) \] (4)

where:
- \( c_i \) - the HPL spectrum of the signal \( x(t) \).

The spectral coefficients \( c_i \) are defined by following relationships:

\[ c_0 = \frac{1}{T} \int_0^T x(t) \, dt \] (5)

\[ c_i = \frac{-1}{2^{k+i}} \int_0^T x(t) \cdot har'(i,t) \, dt \] (6)

where:
- \( har'(i,t) \) – derivatives of continuous Haar functions;
- \( i \) – number of Haar function, \( i = 1, 2, ..., N-1 \);
- \( k \) – group index of the HPL functions,
  \( k = 1, 2, ..., \log_2 N \).

In order to obtain a discrete PWL or HPL functions, the continuous functions are sampled with a constant rate. The obtained set of discrete functions constitutes a basis for the discrete transformations.

As the image itself is a two-dimensional signal, the transformations need expansion into the two-dimensional case. Both transformations are separable, so the two-dimensional expansion is performed by transformations of rows and columns of the image. The resulting two-dimensional spectrum produces an intermediate representation, mapping the image onto the transform domain. The PWL and HPL spectra of an exemplary test image Cameraman are presented in Figure 1.
The presented piecewise-linear spectra show some specific features, which may suggest the selection of the compression method. In both of them polygonal zones of coefficients are visible. The HPL spectrum contains scaled and translated versions of the original, what resembles the spectrum obtained with the use of the Wavelet Transform.

2 DESCRIPTION OF THE APPLIED COMPRESSION METHODS

Transform compression methods use the selected transformation in the first step of the compression algorithm. The essential compression is performed over the obtained spectrum.

Two compression methods are considered at this stage:

– the threshold compression method,
– the zonal compression method.

In general the main aim of both compression methods is to eliminate some spectral coefficients in order to reduce the size of the spectrum. The eliminated coefficients are neglected and their values are set to zero. The elimination algorithm depends on the rule adopted by the chosen method.

In the threshold compression method the spectral coefficients, whose values do not exceed the assumed threshold, are neglected and set to zero. The compression threshold is set arbitrarily, on the base of the initial information on the spectrum characteristics. During the experiments, the compression threshold, being the independent variable, has been assumed as a percentage of the maximum value of the spectral coefficients.

In the zonal compression method only those spectral coefficients, which belong to a selected zone are retained. The location, shape and size of the selected zone are chosen dependently on the spectrum characteristics. In the paper a square zone, placed at the upper left-hand corner of the spectrum, i.e. the one containing the DC coefficient, has been considered. The size of the zone has been assumed as an independent variable.

After the compression process performed in the spectral domain, the resulting spectrum contains many zero-valued coefficients. The reduced spectrum can be further processed, stored or transmitted in the digital image processing system.

The decompression process is performed by calculating the inverse two-dimensional transforms of the compressed spectrum. The reconstructed image is always degraded, as the applied compression methods are lossy, so part of the information is irreversibly lost. The assessment of the reconstruction quality is usually performed with the use of objective measures and subjective visual inspection.

3 RESULTS OF EXPERIMENTS

The above-described compression algorithm has been applied to a representative set of grayscale images. The images are of 256x256 pixels size, with the coding resolution 8bit per pixel. Both PWL and HPL transforms have been applied in the first step of the compression algorithm. The threshold
and zonal compression has been performed. In the decompression process appropriate inverse two-dimensional PWL and HPL transforms have been applied.

As the compression algorithm utilizes different transforms and compression methods, the comparison of its effectiveness requires an objective quality criterion. The reconstructed image quality has been evaluated with the use of typical objective quality measures, as Mean Square Error, $MSE$, and Peak Signal-to-Noise Ratio, $PSNR$, given by the following equations [7]:

\[
MSE(I, \tilde{I}) = \frac{1}{M \cdot N} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (I(i, j) - \tilde{I}(i, j))^2
\]

\[
PSNR(I, \tilde{I}) = 10 \cdot \log_{10} \frac{(L-1)^2}{MSE(I, \tilde{I})} \quad [\text{dB}]
\]

where:

- $I(i,j)$ – original image;
- $\tilde{I}(i,j)$ – reconstructed image;
- $i = 0, 1, ..., M - 1, j = 0, 1, ..., N - 1$;
- $M, N$ – dimensions of the image;
- $L$ – number of grey levels in the original image.

In comparative analysis the considered quality measures are referred to the same criterion – Compression Ratio, $CR$, defined as follows:

\[
CR(I) = \frac{z \cdot 100\%}{M \cdot N}
\]

where:

- $z$ – number of zero coefficients in the image spectrum;
- $M, N$ – dimensions of the image spectrum.

During the experiments the compression quality has been analysed according to various criteria. First, the selected compression method has been applied to both considered transforms. A comparison has been done from the transform point of view. Then, for a selected transform, both compression methods have been applied. The comparison has been done from the compression method point of view. The obtained experimental results may suggest the best combination of transform and compression method.

Figures 2a) and 2b) depict the results of threshold and zonal compression, respectively, with the use of PWL and HPL transforms. The Peak Signal-to-Noise Ratio $PSNR$ has been presented versus the Compression Ratio $CR$. Both selected charts refer to the same test image, $Cam\text{eraman}$, in order to simplify the comparisons.

The results obtained for the threshold compression are more difficult to be analyzed. The lines representing PWL transform (solid line) and HPL transform (dotted line) cross each other at the point of $CR \approx 87\%$, with the corresponding $PSNR$ value equal to 23 dB. For the smaller Compression Ratio values the HPL transform shows better performance, while for the greater $CR$s the PWL transform takes the advantage.

The analysis of the zonal compression results is much easier. Within all the considered range the PWL transform shows a better performance than the HPL one. The difference is constant and equal to about 2dB. It is worth mentioning that in all considered cases the $PSNR$ values show acceptable level, for the assumed reasonable Compression Ratio values.
Another comparison is presented in Figure 3. Part 3a) of the figure shows the compression results with the use of the PWL transform, for threshold and zonal method, while part 3b) presents analogical charts for the HPL transform.

It is clear that in all the considered cases the threshold method shows better performance, provided that the Compression Ratio $CR$ keeps its values within the reasonable range. For the PWL transform the differences are rather slight, on the level of about 1 dB. In the case of the HPL transform the differences are larger; for the first $CR$ value considered in the zonal case, $CR = 75\%$, the difference between the two methods is about 8 dB. It suggests that for both transforms rather the threshold compression method should be recommended.

**CONCLUSIONS**

Effectiveness of the threshold and zonal compression with the use of piecewise-linear transforms has been the subject of investigation. The analysed image has been transformed with the use of PWL or HPL transform, and the essential compression has been performed in the spectral domain. The threshold or zonal methods have been applied. The inverse piecewise-linear transformation applied to the degraded spectrum has led to the reconstructed image. The reconstructed image, degraded by the compression process, has been compared with the original. The compression quality has been
evaluated in terms of objective quality measures. The comparison has been done both from the transform and the compression method point of view. Both threshold and zonal compression methods applied to the piecewise-linear transformed image show good results in terms of the considered measures. In threshold compression the HPL transform shows a better performance, while in the zonal compression the PWL transform gives better results. From the transform point of view, the threshold compression shows higher effectiveness in all the analysed cases. Having analysed the experimental results it may be stated that the considered piecewise-linear transforms, as PWL and HPL, constitute an important alternative for the classical transforms. Application of other methods, as neuro-fuzzy approach [3] in the field of image processing may be the subject of further research.

Abstract

Effectiveness analysis of threshold and zonal compression with the use of piecewise-linear transforms has been investigated in the paper. The piecewise-linear transformations, used in the initial stage of compression algorithm, have been considered. Two transformations have been taken into account: the Periodic Walsh Piecewise-Linear (PWL) Transform and the Haar Piecewise-Linear (HPL) Transform. The transformations have been presented and the expansion into two-dimensional case has been explained. The essential compression has been performed in the PWL or HPL spectral domain. The compression algorithm has been described. In order to evaluate the compression effectiveness, the reconstructed image has been compared with the original one. Computational results of the compression quality have been evaluated in terms of typical quality criteria, as Peak Signal-to-Noise Ratio, PSNR and Mean Square Error, MSE. Results of experiments have been enclosed in the form of plots prepared in the Matlab environment.

Streszczenie

W artykule badano efektywność kompresji progowej i strefowej z zastosowaniem transformat odcinkowo-liniowych. Rozważono transformaty odcinkowo-liniowe, stosowane we wstępnym stadium algorytmu kompresji. Pod uwagę wzięto dwa przekształcenia: okresowe przekształcenie odcinkowo-liniowe Walsha (PWL) oraz przekształcenie odcinkowo-liniowe Haara (HPL). Przedstawiono same przekształcenia i wyjaśniono sposób ich rozwinięcia na przypadek dwuwymiarowy. Zasadnicza kompresja realizowana była w dziedzinie widmowej przekształcenia PWL oraz HPL. Opisany został algorytm kompresji progowej i strefowej. W celu oszacowania efektywności kompresji, obraz rekonstruowany był porównywany z oryginałem. Obliczeniowe rezultaty jakości kompresji zostały wyznaczone z użyciem typowych kryteriów jakości, takich jak maksymalny współczynnik sygnał-szum PSNR oraz błąd średniokwadratowy MSE. Wyniki badań załączono w postaci wykresów opracowanych w środowisku Matlab.

BIBLIOGRAFIA