Analysis of location of distribution centers for scattered construction sites

Introduction

The objective is to analyse and assess the benefits of correct location of warehouses for construction sites and accommodation bases for employees for the project described in the previous article entitled, [13], “Logistic Elements in the Project Involving the Construction of Over 600 Meteorological and Measurement Stations”.

The establishment of the complex system to streamline and improve monitoring, forecasting and warning against dangerous natural phenomena, with the financial aid offered by the World Bank, the project “Monitoring and Country Protection System” (SMOK), [6, 8, 14], involved the construction of over 1000 automatic telemetric measurement stations, scattered across Poland, executed in a cycle of slightly over two years.

Such an untypical project required the establishment of a network of field warehouses with accommodation bases nearby, as working teams travelled daily with the equipment and materials collected from warehouses using entrusted vehicles to construction sites scattered in the area.

According to the principles for model determination of the optimal location for distribution centres, according to the minimum cost and lowest transport activity criteria, the solution of the numerical example points to theoretical locations and assesses the impact of the actual deviation in the location of the field warehouse in Leżajsk. Next, the analysis involved executive complications in the functioning of warehouses established in four regions, respectively with 16, 8, 4, and 8 field warehouses with accommodation bases, and one central warehouse, for the total of 687 stations built.

Location of distribution centres

The location of distribution centres, particularly warehouses, accommodation bases and other auxiliary production background, including in the construction industry, proves a rather complex issue. In the case of making decisions on locating the centre, apart from determining the largest economically justified transport distance between the base location to the target site, [1, 2, 4, 7], there is a problem of determining the location of the distribution centre against the field points of demand, e.g. construction sites using the materials.

When analysing the variants for distribution centre location against the points of demand, one can obtain the solution according to:
- lowest transport activity criterion, used in cases of using similar means of transport and similar transport and field conditions, or
- the cost criterion, applied in the case of using non-uniform means of transport, or when transporting materials in different transport of field conditions.

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The lowest transport activity criterion

Optimal location of the distribution centre, according to the lowest transport activity criterion, at points of demand \( i, i = 1, 2, \ldots, n \), forming the set \( i \in I \), can be defined with the function of the objective, [1, 2, 4]:

\[
\min : \ Z = \sum_{i=1}^{n} q_i l_i
\]  

(1)

where:

\( \min: Z \) – the lowest transport activity criterion, \([\text{deliveries}^\ast \text{km}]\);

\( q_i \) – volume of demand at the point of demand \( A_i \), \([\text{deliveries}]\);

\( l_i \) – distance of the point of demand \( A_i \) from the distribution centre \( A_0 \), \([\text{km}]\).

According to Fig. 1, in the case of locating the distribution centre at point \( A_0 \) with Euclidean record of distance \( l_i \) to particular points of demand \( A_i \), the following is obtained:

\[
\min : \ Z = \sum_{i=1}^{n} q_i \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}.
\]  

(2)

Fig. 1. Location of distribution point \( A_0 \) and the points of demand \( A_i \) in a rectangular system of coordinates.

Source: own study

The lowest transport activity criterion is met where the values of the first derivatives of function (2) against \( x \) and \( y \) are equal to zero, hence:

\[
\frac{dZ}{dx} = \sum_{i=1}^{n} \frac{q_i (x_0 - x_i)}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}} = 0,
\]  

(3)

and

\[
\frac{dZ}{dy} = \sum_{i=1}^{n} \frac{q_i (y_0 - y_i)}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}} = 0.
\]  

(4)

After solving the system of equations with two unknowns, the obtained values \( x_0 \) and \( y_0 \) accurately set the optimal location of point \( A_0 \) meeting the condition of the lowest transport activity criterion. In the circumstances pointed to below, the calculations can be simplified, [4].
In the event of two points of demand $A_1$ and $A_2$ with demands, respectively, $q_1 > q_2$, distribution centre should be located near $A_1$, as it is characterised with greater demand. In turn, in the case of equal demand volumes $q_1 = q_2$, the centre can be located both near point $A_1$ and $A_2$, or at any place of the section linking such points.

In a situation where all points of demand $A_i$ are located on one transport route (in a straight line), the calculations can be simplified by determining the value of the expression $\frac{1}{2} \sum_{i=1}^{n} q_i$. In such a case, the distribution centre should be located near point $i^*$, for which – as the first one when summing up consecutive demand values – the inequality is met:

$$q_1 + q_2 + \ldots + q_i \geq \frac{1}{2} \sum_{i=1}^{n} q_i, \quad i^* \in I.$$ \hspace{1cm} (5)

In turn, in the event of one point of demand $i^*$, with the demand greater than the summary demand of all other points $i$, namely where:

$$q_{i^*} > q_1 + \ldots + q_{i} \ldots + q_n, \quad i \neq i^*, \quad i^* \in I, \quad i \in I,$$ \hspace{1cm} (6)

the distribution centre should be located near point $i^*$.

The locations determined acc. to formulas (3 - 6) still require adjustments, including to the existing local limitations and transport route conditions.

**The cost criterion**

In the case of using non-uniform means of transport, or when transporting materials in different transport of field conditions, the cost criterion is applied, with the function of the objective, \[4\]:

$$\min : \ K = \sum_{i=1}^{n} k_i q_i l_i,$$ \hspace{1cm} (7)

where:
- min. $K$ – the lowest transport cost criterion, [PLN];
- $k_i$ – unit transport cost, 1 tonne (or 1 delivery), [PLN/(t*km), sometimes PLN/(delivery*km)];
- $q_i, l_i$ – as above.

After substitution $K_i = k_i q_i$, the function of the objective has the following form:

$$\min : \ K = \sum_{i=1}^{n} K_i l_i,$$ \hspace{1cm} (8)

The extreme of function $K$ for the lowest costs of transport $K = K_{\min}$ takes place at zero values of derivatives:

$$\frac{dK}{dx} = \sum_{i=1}^{n} K_i \frac{x_0 - x_i}{|l_i|} = 0,$$ \hspace{1cm} (9)

$$\frac{dK}{dy} = \sum_{i=1}^{n} K_i \frac{y_0 - y_i}{|l_i|} = 0.$$ \hspace{1cm} (10)
Example

The construction of meteorological-and-measurement stations in the SMOK project was planned with accommodation bases near the field warehouses, as teams of workers, with equipment and materials collected from the warehouses, used entrusted vehicles to daily travel to particular construction sites scattered in the area.

In order to determine the coordinates of the most favourable (theoretical) location of the distribution centre, due to minimum transport costs, Table 1 lists the data necessary for the actual location of the warehouse and accommodation base in Leżajsk, planned to support 27 construction sites, scattered in the nearby area, with four different station types: ATSH, ATSG, ATSO, ATSM (cf. [13]).

In order to calculate the coordinates of the most favourable location of the distribution centre so that the construction of meteorological

Table 2. Data for the example

<table>
<thead>
<tr>
<th>Station construction site ( i )</th>
<th>( x_i ) [km]</th>
<th>( y_i ) [km]</th>
<th>Demand ( q_i ) [deliveries]</th>
<th>Unit cost of delivery ( k_i ) [PLN]</th>
<th>( K_i ) [PLN]</th>
<th>Distance from Leżajsk to the site ( l_{Li} ) [km]</th>
<th>Duration of transport from Leżajsk to the site ( t_{Li} ) [h]</th>
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<td>50</td>
<td>4.70</td>
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</table>

When applying the lowest transport cost criterion, occurring for the values of first derivatives against \(x\) and \(y\), acc. to formulas (9 and 10) equal to zero, with the solution on the basis of Mathcad software and when applying the data from Table 1, the optimal location of the distribution centre is at point \(A_0(x_0, y_0)\):

- with coordinates, \(x_0 = 63.58833\) km, \(y_0 = 48.88546\) km,
- function of the objective, \(K_{\text{min}} = 8178.85\) PLN,
- derivatives, \(P_x = -4.3E-08\), \(P_y = -5.8E-08\).

For the purpose of comparing calculation results, while using formulas (3 and 4), at point \(A_0^+(x_0^+, y_0^+)\), another location of the distribution centre has been determined, using the lowest transport activity criterion. When using the data from Table 1 and Mathcad software, the following results have been obtained:

- coordinates of point \(A_0^+(x_0^+, y_0^+)\), \(x_0^+ = 64.38752\) km, \(y_0^+ = 49.01401\) km,
- function of the objective, \(K_{\text{min}}^+ = 4425.23\) PLN,
- derivatives, \(P_x^+ = -9E-08\), \(P_y^+ = 1.99E-08\).

When analysing the location of distribution centres acc. to the lowest transport cost and the lowest transport activity criteria, it can be noticed that their optimal locations, respectively in points and \(A_0(63,59,48,89)\) and \(A_0^+(64,39, 49,01)\) are not far distant from one another (Fig. 2b; distance from the light dot to the cross). In the analysed case, there is small impact of variability of unit costs of delivery \(k_i\).
The costs vary for particular stations from 1.71 to 2.09 PLN/(delivery*km), namely are higher within the range up to 22% against the lowest value. Due to non-orderly occurrence of such differing cost values, their actual impact on the centre location can be mutually balanced. Therefore, in the analysed case, the most favourable locations are only slightly distant by approximately 0.713 km.

Actually, for the analysed stations, the warehouse and the accommodation base were located in Leżajsk. The main cause of adopting such a location were the favourable lease conditions of the warehouse with the adjacent square, and easy organisation of the accommodation base for employees. Leżajsk is far from the most favourable location in the cost aspect, namely from the theoretical distribution centre $A_0(63,59,48,89)$ by approximately 19.5 km. The scattering of construction sites in the area, according to road distances, ranged from 8 km to 96 km, and their average distance from the warehouse and accommodation base totalled 54 km.

The network of warehouses and accommodation bases for employees

On the basis of the analyses and actual conditions regarding technical location capabilities when executing the SMOK project, a warehouse and accommodation network was established on the territory of Poland, comprising the central warehouse in Krakow and 35 separate units grouped in four regions (Fig. 3).

Locations and the number of field warehouses depended on the intensity of station distribution in a particular area and technical possibilities of effective (quick) establishment of a warehouse and assuring the accommodation base for the workers. As a result, the following were established [11]: in the Krakow Region - 16 units (namely field warehouses with accommodation base nearby), in the Poznan Region - 4 units, in the Wroclaw Region – 8 units, and in Gdynia Region – 7 units.

According to the delivery schedule adjusted by untimely delivered particular types of prefabricated elements, the coordinating team in charge of the project logistics, [5, 7, 10, 12], considering the production
capacity of the supplier and progress of works at the construction sites, ordered the necessary quantities of specialist materials every week, according to the assortments available from the main supplier-manufacturer, with the instruction of full-vehicle deliveries to particular field warehouses.

Standard and generally available materials did not pose problems, they were ordered in wholesale volumes, usually from local suppliers, and possible shortages were supplemented on continuous basis in retail stores.

The central warehouse in Krakow had the intervention function, and was intended for materials remaining after completed stations, [13]. The warehouse had a minimum reserve of the materials most in deficit, which could be distributed on intervention base to the delayed sites.

The insufficient production capacity of prefabricated elements to be built-in required systematic coordination of the progress of works at the sites and the work of particular teams as compared to the possibility of supplying the necessary elements.

In a situation of the occurring delays in production, particularly regarding the masts, the central warehouse in Krakow had the intervention function regarding procurement. While having a minimum reserve, it provided single item deliveries of missing materials to the most delayed sites. In emergency situations, it organised dispatch of a full load according to the route planned, covering even several hundred kilometres, leaving the necessary material items at particular sites.

**Conclusion**

In the case of making decisions on locating the distribution centres, apart from determining the largest economically justified transport distance, there is a problem of determining the location of the distribution centre. A solution can be obtained according to the criterion, namely the lowest transport activity or according to the cost criterion.

When analysing, in the example, the location of the distribution centre being a warehouse with the accommodation base nearby, to support 27 construction sites of meteorological and measurement stations scattered in the area, remaining in the range of from several to approximately 100 km, the most favourable locations of the distribution centre were set according to two criteria. While using the Mathcad calculation software, it was determined that the centre location points, according to the lowest costs of transport and the lowest transport activity, are not much distant from one another, just by about 0.713 km. Such a result was due to, among others, small variability of unit costs of deliveries (from 1.71 to 2.09 PLN/(delivery*km)) and their unordered impact which, with the occurring vectors of the location of stations scattered in the area, can mutually balance them. Actually, due to favourable financial terms of warehouse rental and easy organisation of the accommodation base, Leżajsk was selected as the location, distant by approximately 19.5 km from the optimal location determined due to the lowest costs of transport.

In a situation of too low efficiency of production of prefabricated elements to be built-in, it was necessary to have a coordination team in charge of logistics of the project who systematically adjusted the progress of works at the sites and the work of particular teams as compared to the possibility of supplying the necessary elements.

The central warehouse in Krakow, while rationally managing the minimum reserve, assured single item deliveries of missing materials in emergency situations, organising intervention dispatch of full loads, according to the planned route to particular construction sites.

**Analiza lokalizacji centrów dystrybucji środków dla rozproszonych budów**

**Streszczenie**

Przedsięwzięcie związane z wybudowaniem w dwuletnim cyklu, ponad 1000 automatycznych stacji meteorologiczno-pomiarowych, rozproszonych na obszarze Polski, realizacyjnie niestandardowe, było równocześnie ważnym i złożonym zadaniem logistycznym.

W kontekście modeli teoretycznych lokalizacji centrów dystrybucji względem punktów zapotrzebowania, przeanalizowano rzeczywiste umiejscowienie magazynu z bazą noclegową w sąsiedztwie, dla po-
trzeb realizowanych budów rozproszonych w terenie. Przedstawiono przykład obliczeniowy z najkorzystniejszymi lokalizacjami, przy kryteriach minimum kosztów transportu oraz minimum pracy transportu, wraz z analizą otrzymanych wyników, jak też rzeczywiste usytuowania magazynu w Leżajsku, obsługującego 27 budów, rozproszonych w promieniu około 96 km.

Przeanalizowane zostały problemy przy funkcjonowaniu magazynów terenowych w 4 regionach projektu SMOK, tj. w Krakowie, Wrocławiu, Poznaniu i Gdańsku, które obsługiwały razem 687 budowanych stacji. Wskazano również korzyści interwencyjnego funkcjonowania magazynu centralnego w Krakowie.

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