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Determining the coverage zone and accuracy zone of the azimuth system – in the past and today

INTRODUCTION

From the very beginning, people have been travelling and exploring new territories, and each time there was a need for recording one’s route and discoveries so as to be able to return to the place one had found and also to boast about one’s achievements. Even ancient people felt the need to record what the area around them looked like. At the beginning they did this on cave walls. The oldest wall painting, which was discovered in Çatal Hüyük, dates from around 6500 B.C. and presents a settlement plan with a nearby volcano [8][3].

Fig. 1. The oldest known topographic map in the form of a tablet from eastern Mesopotamia (Iraq), dating back to around 3000 B.C. [4].

At subsequent stages of human evolution, maps and charts as we know them today were created; they differ depending on their intended use. Among the oldest Polish maps and charts is the nautical chart of the western part of the Bay of Gdańsk (Figure 2).

Fig. 2. Nautical chart of the western part of the Bay of Gdańsk from 1596 [2].

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3 Artykuł recenzowany.
In order to travel, people not only needed to use maps or charts, but they also had to identify their location relative to the representation of a given area on maps and charts as well as to orient themselves in their surroundings. Therefore, since the time people began travelling, they have been looking for distinctive objects that would help them to safely return to the place from which they set out. Because it was not always possible to use natural landscape elements, people started to place navigation marks, which are used for sea travel, along the shore. Thus, the era of navigation marks began. A lighthouse that was built on the island of Pharos around the year 280 B.C. was the most famous one. It was 160 metres high and probably had a light range of approx. 50 km; it became a model for most other lighthouses (Figure 3).

![Lighthouse on the island of Pharos](image)

**Fig. 3.** Lighthouse on the island of Pharos [1].

These were the beginnings of the oldest type of marine navigation, i.e. terrestrial navigation. Until the end of the 19th century, terrestrial navigation was based on optical navigation systems, and currently it is considered the second most often used type of navigation aids after satellite systems as far as coastal sailing is concerned. Since there are many navigation marks on land which have different sizes and colours, navigators do not always notice every mark while watching the shore, which is good because navigators should only receive selected nautical information. Navigators never need all navigation marks; they only need to use those which allow them to determine their own vessels’ positions at sea at a given moment and with specific accuracy. Therefore, it is important to plan a sea journey also with regard to the systems of navigation marks that will be used. The accuracy of a vessel’s own position at sea is one of the factors in selecting the appropriate systems of navigation marks. This accuracy is established in relation to the tasks that are performed by vessels. Detailed analyses of the types of accuracy and methods for determining a vessel’s position with particular accuracy are presented in the literature on this topic, for example in [5] and [10]. The coverage zone of navigation marks, i.e. their availability in a given sea area, is another factor that is extremely important in determining which navigation aids should be used. These two factors play a decisive role in establishing whether it is possible to use a particular system of navigation marks to carry out specific navigation tasks in a given sea area. This is why both the coverage zones and the accuracy zones of navigation marks have always been established in the process of designing new systems of navigation marks or modernising the existing ones. The methodology for determining these zones has been known and used for years [7] and [13].

Currently, in an era of rapidly developing mobile computer technology, it is possible to use the methods for determining both coverage and accuracy zones that have been known for years in the process of planning a sea journey or when identifying the available navigation marks during this kind of journey. Therefore, this paper presents the results of a project that was aimed to automate the process of determining coverage and accuracy zones by using a commonly available smartphone that runs on the Windows Phone operating system. A system of lighthouses that are located on the Polish coast was used to test whether the application works correctly; this system is also described in this paper to the extent necessary for the purposes of discussing the application. The text below presents a solution to the problem of automating the process of establishing a two-element system that uses bearings to determine a vessel’s own position.
1. METHODOLOGY FOR DETERMINING THE COVERAGE ZONE AND ACCURACY ZONE FOR A TWO-ELEMENT AZIMUTH SYSTEM

The coverage zone of every system of navigation marks, including the azimuth system, is an area situated within the operating range of two stations (navigation marks) where one can determine a vessel’s position. The size of the azimuth system’s coverage zone depends on the following elements [13]:

- type of the navigation system,
- coverage zones of coastal radio stations \( d_A, d_B \),
- geometrical arrangement of coastal radio stations,
- assumed minimum accuracy (value) with which a vessel’s position is determined.

The boundaries of the coverage zone are determined by using the limit value of the angle at which lines of position \( \theta \) intersect as well as the coverage zone (visibility range) of coastal radio stations \( d_s \). Both these parameters can be used together or separately.

Fig. 4. Coverage zone of the azimuth system.

Each coverage zone is characterised by:

- operating range \( d_s \), which is calculated starting from a coastal radio station up to the most distant boundary point and which is based on the formula:

\[
d_s = \frac{b}{2 \tan \left( \frac{\Theta}{2} \right)}
\]  
(1)

- coverage radius \( r \):

\[
r = \frac{b}{2 \sin \Theta}
\]  
(2)

- area of the coverage zone \( \Theta_s \):

\[
Q_s = \pi - r^2 (2\Theta - \sin 2\Theta)
\]  
(3)

The accuracy zones of systems of navigation marks are identified by determining the geometric loci of points at which a vessel’s position can be established with equal accuracy within a given coverage zone. An accuracy zone is described by an area that corresponds to the minimum and maximum accuracies as well as intermediate areas between these values.
The methodology for determining the accuracy zone of the azimuth navigation system amounts to:

1. Calculating the maximum value of a position error \( M_{\text{max}} \) for the assumed minimum value of angle \( \Theta \):

\[
M_{\text{max}} = \sigma_{NR} \cdot \arcsin \left( \frac{D_{\text{max}} \sqrt{2}}{\sin \Theta} \right)
\]  

(4)

where:

\[
D_{\text{max}} = \frac{b}{2 \sin \left( \frac{\Theta}{2} \right)}
\]  

(5)

2. Calculating the minimum value of a position error \( M_{\text{min}} \) by using the formula:

\[
M_{\text{min}} = \frac{0.0125 \cdot b \cdot \sigma_{NR}^2}{\sin \Theta \cdot \sin \left( \frac{\Theta}{2} \right)}
\]  

(7)

3. Determining intermediate values \( M_i \) that satisfy the condition: \( M_{\text{min}} < M_i < M_{\text{max}} \) by using the formula:

\[
M_i = \sigma_{NR} \cdot \arcsin \left( \frac{d_A^2 + d_B^2}{\sin \Theta} \right)
\]  

(8)

For each value of a mean error that has been determined, the geometric factor \( k \) is calculated:

\[
k = \frac{M}{b \cdot \sigma_{NR}}
\]  

(6)

Then, by using Table 1, pairs of angles \( Z_1 \) and \( Z_2 \) are determined for factor \( k \) that has been calculated.

After selecting pairs of angles that satisfy the condition: \( k = f \left( Z_1, Z_2 \right) = \text{const} \), an accuracy isoline for a given value of error \( M \) is drawn on a map.

An error ellipse for equal accuracy lines is one of the measures of accuracy that are used in navigation. Therefore, when determining the accuracy zone of a two-element azimuth navigation system, one should also determine the parameters of the ellipse, i.e. its semi-axes \( (a, b) \) and angle of rotation \( (\varphi) \).

In order to do this, one must first determine the value of the mean error of position lines by using the following formula:

\[
\sigma_{lp} = \arcsin \cdot \sigma_{NR} \cdot D
\]  

(9)

When the values of the mean errors of position lines \( (\sigma_{lp_1}, \sigma_{lp_2}) \) as well as the angle \( (\theta) \) are known, one should determine mean vector errors \( V_1 \) and \( V_2 \), which constitute conjugate semi-axes of the mean error ellipse.
Then the values of the ellipse’s semi-axes should be determined:

\[ a + b = \sqrt{V_1^2 + V_2^2 + 2V_1V_2 \sin \Theta} \quad (12) \]
\[ a - b = \sqrt{V_1^2 + V_2^2 - 2V_1V_2 \sin \Theta} \quad (13) \]

**Tab. 1. Values of geometric factor** $k$ (source: [Smirnovskij A.F. 1967]).

![Table 1](image)

**Fig. 6.** Mean error ellipse.

The angle of rotation ($\varphi$) between the semi-major axis of the ellipse and the larger vector error $V_1$ can be determined by using the formula:

\[ \tan 2\varphi = \frac{\sin 2\Theta}{\left(\frac{V_1}{V_2}\right)^2 + \cos 2\Theta} \quad (14) \]

By plotting the angle of rotation ($\varphi$) from the larger vector error one can determine the direction of the semi-major axis ($a$), which is plotted from a vessel’s position in the interior of the angle of intersection between position lines ($\Theta$) with arms $V_1$ and $V_2$. 

The above methodology for determining the coverage zone and the accuracy zone was used in an application which is described later in the paper.

2. SYSTEM OF LIGHTHOUSES IN POLISH SEA AREAS

The authors decided to use one of the subsystems of navigation marks that are located in Polish sea areas to test whether the application they had created was functional and worked properly. According to [Act of 1991], the term ‘Polish sea areas’ refers to Polish internal sea waters, territorial sea and exclusive economic zone (Figure 7). The Polish coast extends between the meridians $\lambda = 014^\circ 13,0^\prime$ and $\lambda = 019^\circ 38,0^\prime$ E. The dolphin “Dalba 7” marks Poland’s border with the Federal Republic of Germany. It is located on Łysa Góra’s foreland ($\varphi = 53^\circ 44.3^\prime$ N, $\lambda = 014^\circ 16.4^\prime$ E) and has an arc of a light sector of $153^\circ - 010^\circ$. Poland’s border with the Russian Federation is marked by leading marks (leading lights) that indicate a direction of $136.5^\circ$. The front (lower) light is located at a position with coordinates $\varphi = 54^\circ 27.4^\prime$ N, $\lambda = 019^\circ 38.4^\prime$ E, whereas the rear light at a position with coordinates $\varphi = 54^\circ 27.2^\prime$ N, $\lambda = 019^\circ 38.8^\prime$ E.

Fig. 7. Polish sea areas [3].

Tab. 2. Lighthouses on the Polish coast [BHMW 2010].

<table>
<thead>
<tr>
<th>Lighthouse</th>
<th>Geodetic coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swinoujście</td>
<td>$\varphi = 53^\circ 55.0^\prime$ N, $\lambda = 014^\circ 17.2^\prime$ E</td>
</tr>
<tr>
<td>Kikut</td>
<td>$\varphi = 53^\circ 59.0^\prime$ N, $\lambda = 014^\circ 34.9^\prime$ E</td>
</tr>
<tr>
<td>Niechorze</td>
<td>$\varphi = 54^\circ 05.8^\prime$ N, $\lambda = 015^\circ 03.9^\prime$ E</td>
</tr>
<tr>
<td>Kołobrzeg</td>
<td>$\varphi = 54^\circ 11.3^\prime$ N, $\lambda = 015^\circ 33.4^\prime$ E</td>
</tr>
<tr>
<td>Gąski</td>
<td>$\varphi = 54^\circ 14.7^\prime$ N, $\lambda = 015^\circ 52.5^\prime$ E</td>
</tr>
<tr>
<td>Darłowo</td>
<td>$\varphi = 54^\circ 26.5^\prime$ N, $\lambda = 016^\circ 22.8^\prime$ E</td>
</tr>
<tr>
<td>Jarosławiec</td>
<td>$\varphi = 54^\circ 32.5^\prime$ N, $\lambda = 016^\circ 32.7^\prime$ E</td>
</tr>
<tr>
<td>Ustka</td>
<td>$\varphi = 54^\circ 35.4^\prime$ N, $\lambda = 016^\circ 51.4^\prime$ E</td>
</tr>
<tr>
<td>Czołpino</td>
<td>$\varphi = 54^\circ 43.2^\prime$ N, $\lambda = 017^\circ 14.6^\prime$ E</td>
</tr>
<tr>
<td>Sîło</td>
<td>$\varphi = 54^\circ 47.3^\prime$ N, $\lambda = 017^\circ 44.2^\prime$ E</td>
</tr>
<tr>
<td>Rozewie</td>
<td>$\varphi = 54^\circ 49.9^\prime$ N, $\lambda = 018^\circ 20.3^\prime$ E</td>
</tr>
<tr>
<td>Jastarnia</td>
<td>$\varphi = 54^\circ 42.1^\prime$ N, $\lambda = 018^\circ 41.1^\prime$ E</td>
</tr>
<tr>
<td>Hel</td>
<td>$\varphi = 54^\circ 36.1^\prime$ N, $\lambda = 018^\circ 48.9^\prime$ E</td>
</tr>
<tr>
<td>Sopot</td>
<td>$\varphi = 54^\circ 26.8^\prime$ N, $\lambda = 018^\circ 34.4^\prime$ E</td>
</tr>
<tr>
<td>Port Północny</td>
<td>$\varphi = 54^\circ 24.1^\prime$ N, $\lambda = 018^\circ 41.9^\prime$ E</td>
</tr>
<tr>
<td>Krynica Morska</td>
<td>$\varphi = 54^\circ 23.2^\prime$ N, $\lambda = 019^\circ 27.2^\prime$ E</td>
</tr>
</tbody>
</table>

Many types of systems of navigation marks can be used for navigation in Polish sea areas. The GPS and GLONASS satellite systems are among the most common ones. However, it is classical systems of navigation marks, such as radar or optical systems, that are still used in the coastal zone when approaching a port. The system of lighthouses that are located along the entire Polish coast is a
subsystem of the system of visual aids to navigation. There are currently 16 operational lighthouses on the Polish coast. This subsystem constitutes easily identifiable two-element azimuth systems, which is why it was considered to be sufficient to test the mobile application.

3. AUTOMATING CALCULATIONS BY USING A MOBILE APPLICATION

Since mobile devices are commonly available and they are being constantly developed, it is possible to automate and ensure continuity of the provision of information about the availability of navigation systems as well as other types of nautical information. The mobile application that has been created and that allows one to perform the task mentioned in the title of this paper is an example of how calculations can be automated. The programming of applications for modern mobile phones, which are becoming increasingly comparable with computers in terms of the available memory and computing power, resembles the process of programming computer applications. The authors of this paper have developed an application in the Windows Phone environment in standard C#.

In order to accurately determine the elements of coverage and accuracy zones, including base line B, coordinates B, L were transformed into a two-dimensional rectangular coordinate system (X,Y). It is widely known how to carry out this transformation, which has been widely described in the literature on the subject and in normative geodetic documents. This transformation can also be performed by using programs that are available on the Internet, for example, [4], [5] and [6]. The authors used the mathematical relationships published in [5], which made the process of carrying out determinations in the application easier.

The text below presents a short description of how this application can be used:
– In order to start this application, one should select the “Calculating zones” icon (Fig. 8); after opening the application, one will see a screen (Fig. 9) showing the first page, where data on two navigation marks should be entered.

![Fig. 8. Starting the application.](image)

![Fig. 9. Screen for entering data on a two-element azimuth system.](image)
- The data can be either entered manually or selected from a list of available lighthouses (the “select data from the list” button – Fig. 10).
- After touching the “select data from the list” button and when selecting the second navigation mark, only those lighthouses will appear which can constitute a two-element system together with the lighthouse that was selected first; for example, if the Świnoujście lighthouse was selected first, one will see the Kikut lighthouse appear in the second window (Fig. 11.).
- After moving the screen up, one will see two parameters: the minimum angle at which position lines ($\Theta$) intersect and the value of an error in determining the bearing ($\sigma_{NR}$) [11]; if the default values do not suit the user, he/she can enter his/her own parameters (Fig. 12).

![Fig. 10. Selecting the first lighthouse.](image)

![Fig. 11. Selecting the second lighthouse.](image)

![Fig. 12. Entering additional parameters.](image)

- After entering the data and touching the “Calculate” button, one obtains elements of the coverage and accuracy zones of the azimuth system (Fig. 13); the parameters that have been determined are used to draw zones on a map.
CONCLUSIONS

1. The methodology for determining the coverage zones and accuracy zones of two-element azimuth navigation systems has been known and used for years in the process of providing navigational and hydrographic support for human activity at sea. The widespread availability of computation technology stimulates the development of the known methods for providing this support and helps to identify new applications of these methods. This is also true of the method for automating the process of determining the coverage zones and accuracy zones of navigation systems that is described in the present paper.

2. This method allows the navigator to perform typical navigational tasks in a new way, i.e. more quickly and independently. The problem that is discussed here constitutes an example of how a mobile application can be used in the process of planning a sea journey and selecting navigation marks in order to be able to conduct observations. Until recently, this methodology was only used by teams that designed new navigation systems and modernised the existing ones, whereas the application that has been developed by the authors allows one to quickly and intuitively use this methodology during a sea journey.

3. The kernel of this software makes it possible to freely add new applications and modify the existing ones if they are of use to the navigator and can be installed on devices such as smartphones or tablets.

Abstract

Methods for determining the coverage zones and accuracy zones of navigation systems have been known for years and they have been used in the process of providing navigational and hydrographic support for human activity at sea. The problem that is most often analysed involves defining the availability of two-element navigation systems and the quality of determinations that are carried out based on these systems. The changing technical capabilities as well as the need to have the most recent and continuous navigational information makes it necessary to use new, commonly available electronic equipment. This paper aims to fulfil these expectations and, therefore, it recapitulates the methodology of determining the coverage zone and accuracy zone of a two-element azimuth system and presents one of the possible ways of automating calculations by using mobile equipment.

Określanie obszaru pokrycia i strefy dokładności poziecy w systemie naprowadzania – wczoraj i dziś

Streszczenie

Metody określania strefy zasięgu i dokładności systemów nawigacyjnych są znane i używane w procesie wspierania nawigacyjnej i hydrograficznej aktywności ludzkiej na morzu od lat. Głównym problemem w tym obszarze jest określanie dostępności dwuelementowych systemów nawigacyjnych i dokładności pozyskiwanych przez nie danych. Zmieniające się możliwości techniczne, w omawianym kontekście sprawiają, że konieczne staje się udostępnienie wykorzystywanych dotychczas systemów Nawigacyjnych dla powszechnie stosowanego sprzętu elektronicznego. Artykuł ma na celu podsumowanie wiedzy na temat metod wyznaczania obszaru pokrycia oraz strefy dokładności systemu azymutu dwóch elementów i przedstawienie jednego z możliwych sposobów zautomatyzowania obliczeń przy użyciu urządzenia przenośnego.
REFERENCES

17. http://www.numerus.net.pl/programy.html