INTRODUCTION

Traffic jams, congestion and other types of traffic malfunctions are mainly the result of incorrectly planned infrastructure. There are many badly planned networks of crossroads, including traffic lights settings and number of lanes. It involves a danger of performance degradation that increases exponentially with traffic load. Many studies showed a big impact of the efficiency of transportation system on air pollution [1, 3, 5] as well as economic aspects [11]. One of the approach to cope with different cases of transportation problems (mainly bottlenecks and congestions) is modeling that is widely used because of relatively low cost and flexibility [2, 6, 8, 9].

Classical approach mostly assumes independent and identically distributed random variables that makes all analysis more tractable and often leads to the ability of derivation of the analytical forms of the traffic performance measures. On the other hand, the real traffic is not ideally uncorrelated and introduces long range dependence (LRD). The main difference is that the autocorrelation function decays very slowly and is not summable:

$$\sum_{k=0}^{\infty} r(k, H) \to \infty$$, where

$$r(k, H) = \frac{\sigma^2}{2} \left[ (k+1)^{2H} - 2k^{2H} + |k-1|^{2H} \right]$$ for \( k = 0, 1, \ldots \)

1. MULTI-LANE TRAFFIC MODEL

Multi-lane traffic model is based on a discrete time Markov chain. To obtain its steady state probability distribution the following conditions must be satisfied:
\[
\bar{\pi} = \lim_{k \to \infty} \pi(k) \quad \text{and} \quad \sum_i \pi_i = 1
\]  

(3)

For two-state version, presented in figure 2 (left side), one can obtain:

\[
[\pi_0 \, \pi_1] = [\pi_0 \, \pi_1] \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}
\]

(4)

From (3) and (4), the exact values of \( \bar{\pi} \) are:

\[
\pi_0 = \frac{q}{p+q} \quad \text{and} \quad \pi_1 = \frac{p}{p+q}
\]

(5)

Taking into consideration more states and different connections among them, like in figure 2 (right side), and additionally the assumption that in state 0 every discrete time slot only one vehicle is generated as a part of the whole traffic, one can obtain Markov chain modulated by Bernoulli process \[10\]. It is described by only 3 parameters: \( n, a \) and \( q \), where \( n \) is the number of states \((i = 0, 1, \ldots, n-1)\), \( a \) and \( q \) are the parameters responsible for long range dependency and the mean of the process.

The transition matrix for this process has the following form:

\[
A = \begin{bmatrix}
1 & -\frac{1}{a} & -\frac{1}{a^2} & \cdots & -\frac{1}{a^{n-1}} & \frac{1}{a} & \frac{1}{a^2} & \cdots & \frac{1}{a^{n-1}} \\
\frac{q}{a} & 1 - \frac{q}{a} & 0 & \cdots & 0 \\
\left(\frac{q}{a}\right)^2 & 0 & 1 - \left(\frac{q}{a}\right)^2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\left(\frac{q}{a}\right)^{n-1} & 0 & 0 & \cdots & 1 - \left(\frac{q}{a}\right)^{n-1}
\end{bmatrix}
\]

(6)

Since a vector \( \bar{\pi} \) is considered to be stationary \((\bar{\pi} = \bar{\pi} \cdot A)\), one can obtain many measures such as expected value or variance. An example probabilities for \( q = 0.5766 \) and \( a = 6.51 \) is shown in figure 3. Additionally, modifying \( a \) and \( q \) parameters of the model, one can introduce LRD properties, which will be discussed in the next section.
2. TRAFFIC SYNTHESIS WITH LRD PROPERTY

Long range dependence is widespread in real world, especially in processes where external factors influence independent random nature of theoretical and isolated phenomenon. This applies to many fields, such as for example computer networks, operating systems, process automation, etc. One of the fastest method of synthesis LRD process is the frequency based method of fractional Gaussian noise [7]. Since the method does not provide information of the exact time of single object (vehicle), a method based on Markov chains seems to be better approach. Long range dependence can be observed when events (vehicle occurrences) spaced in time are correlated and the value of $H$ (eq. 2), called Hurst exponent, is inside the range: $H \in (0,5,1)$.

For stationary sequence $X = \{X(i), i \geq 1\}$ the corresponding aggregated sequence with the level of aggregation $m$ can be obtained by averaging over non-overlapping blocks of size $m$:

$$X^{(m)}(k) = \frac{1}{m} \sum_{i=(k-1)m+1}^{km} X_i$$  \hspace{1cm} \text{for } k=1,2,… \hspace{1cm} (7)$$

and the variance of this aggregated random variable tends to:

$$\text{var}(X^{(m)}) \sim \alpha \cdot m^{2H-2}$$  \hspace{1cm} (8)$$

In this Markovian model, the corresponding variance value for aggregated sequence has the analytical form:

$$\text{var}(X^{(m)}) = \frac{E(N)}{m} \left[ 1 + \frac{m-1}{m} \sum_{i=1}^{m-1} (m-1)A_{0,0}^i \right] - E(N)^2$$  \hspace{1cm} (9)$$

and can be used for tuning $\alpha$ parameter of the model for any value of $H$. Since the model is discrete, synthesis of the vehicle (car) traffic can be carried out by the aggregation of single sources, like for example in figure 4. The aggregated process is also LRD and has the same level of self-similarity. Traffic intensity measured as a total number of vehicles on all lanes per unit time can determine the input rate of the input of analyzed queueing system. It follows that the measurement line (figure 1) can be located anywhere on a highway or freeway, close to the point where the analyzed queueing system with the finite buffer space is theoretically placed. However, if the road does not split or merge and the number of lanes remains the same.

Fig. 3. Steady state probabilities of five-state Markov chain
3. RESULTS

The most common way to estimate the level of LRD is the value of Hurst exponent, however its estimation is not straightforward. Even though there are many methods, one can focus on three of them: variance-time, index of dispersion for counts and periodogram.

As it was mention in the previous section, \( a \) and \( q \) values can be calculated using the following methods: variance-time, secants, bisection and linear regression. Values of parameters obtained for \( n \)-state Markovian model by fitting to the analytical variance for this process (eq. 9), especially for higher level of aggregation of random variable (above 1000). Three methods of estimation of LRD level have been used: variance-time (VT), Index of Dispersion for Counts (IDC) and periodogram. The results of estimation are shown in table 1. Values of \( H \) are different for these methods, however they are close to the desired values. One can observe that the higher value of \( H \) the bigger differences among results for these methods. It is quite normal for the high variability related to the structure of the model, as well as for the errors introduced by estimation methods.

![Graph](image-url)

**Fig. 4.** Single lane discrete time occupancies and cumulative values

![Graph](image-url)

**Fig. 5.** Example of four simulated traces for \( H=0.7 \)
Fig. 6. Example of four simulated traces for H=0.8

Tab. 1. Results of Hurst exponent estimation for generated traffic

<table>
<thead>
<tr>
<th>$H$</th>
<th>Estimated $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance-time</td>
</tr>
<tr>
<td>0.70</td>
<td>0.683</td>
</tr>
<tr>
<td>0.75</td>
<td>0.712</td>
</tr>
<tr>
<td>0.80</td>
<td>0.772</td>
</tr>
<tr>
<td>0.85</td>
<td>0.824</td>
</tr>
<tr>
<td>0.90</td>
<td>0.837</td>
</tr>
</tbody>
</table>

Results of analysis of queueing system, where the input stream corresponds to vehicles generated by the 5-state Markov modulated Bernoulli process with rate 3.5 vehicles / s, are shown in figure 7. There is a visible difference between mean number of vehicles for (no LRD property) and for. It can be seen that for the same number of vehicles (say 10) the throughput of the highway/freeway should be increased by tens of percent (for 10 vehicles in the system, 39%).

Fig. 7. Number of vehicles in the queueing system, input rate: 3.5 vehicle/s

CONCLUSIONS

Because of the big impact of LRD on queueing performance, one should take this property into consideration in research analysis, especially in modeling of any processes associated with traffic. Since the Markov modulated Bernoulli process has only three parameters that are responsible for
fitting both intensity and the LRD property of the car traffic, it is very tractable. However, it is suggested to recalculate $a$ values for different $H$, which leads to a more accurate estimates of Hurst exponent. Even small differences in high values of $H$ involve big differences of the mean number of vehicles in the queueing system for the given throughput, or in other words, higher values of throughput (tens of percent) for given number of vehicles in the system (figure 7). Furthermore, one can notice some irregularities in the mean for higher $H$ values, that can be caused by the bursty nature of highly correlated process. On the other hand, this model is very interesting from the point of view of application to queueing analysis because of discrete times and ability to simulate any number of lanes as well as every occurrence of a single vehicle per time unit.

Abstract

This paper investigate modulated Markov chain to model multi-lane traffic. The model focuses on long range dependence that has a big impact on queueing performance and efficiency of cars transportation system. The results of discrete time simulation as well as the effects of the influence of long range dependence on queueing is presented and discussed.

Keywords: traffic modeling, multi-lane traffic, long range dependence

Streszczenie

Artykuł rozpatruje modulowany łańcuch Markova jako model ruchu na drodze wielopasmowej. Model ten skupia się na zależnościach długoterminowych, które mają duży wpływ na efektywności działania kolejkowania w samochodowych systemach transportowych. Rezultaty symulacji dla modelu z czasem dyskretnym, jak również efektywu zależności długoterminowych na kolejkowanie, zostały zaprezentowane i poddane dyskusji.

Słowa kluczowe: modelowanie natężenia ruchu, ruch na drodze wielopasmowej, zależności długoterminowe

REFERENCES