Introduction

Demand reduction and competition are forcing enterprises to improve product quality, searching for new products as well as improving the sales organization. Creating significant market power with strong competition from other players is therefore not only the issue of shortening the delivery time or reducing supplies and ordering cost (Skowronek 2013). The company can also take account of the demand of marketing logistics, such as individualized customer needs (Peck et al. 2006), creating innovative products, building relationships with customers, the appropriate level of service, creating value for the customer, and finally, category management. In all these areas, in the integrated management of logistics and marketing, the company management increasingly adopts the results of modern industrial experiments to find the optimum solution in terms of competition and market segmentation. Modern experimental systems and new methods of data analysis, which are available via computer software require the knowledge about the conditions limiting their application as well as the knowledge about the range and importance of information obtained from statistical analysis. Particularly, analysis of experimental results with repeated measurements is complicated as results from the correlation between the errors of consecutive measures. In this case, the form of covariance matrix is very important and can be very simple when its structure meets the condition of compound symmetry, and complicated (unstructured) when it does not satisfy it. The few parametrical methods of such data have been developed, thus one can choose between univariate analysis for split-plot model in time/space, multivariate approach or analysis of mixed models.

The aim of this paper is the presentation of logistic path of procedure of the choice analyzing method of repeated measurements data due to the nature of these data and information available to the researcher, or the one he wants to obtain. This path is particularly important when using specialized statistical software, which offers a high degree of data analysis at the same time does not provide guidance about the next steps of analysis.

Methods

Specification of statistical methods limits as well as determination of the information range obtained from chosen method is needed to develop the logistic track procedure of the statistical analysis selection. The limitations and the order of proceedings in three basic parametric methods taken into account in the selection procedure will be discussed. The simplest method of the analysis of repeated measurements, but the less effective one, is the data analysis on each level of repeated measures factor separately. Unfortunately, in such a case we lose too much information about the changes in time or space of a studied feature. This is why in this paper we do not take into consideration this kind of analysis.

Univariate analysis for the model “split plot in time”

The term “split plot in time” means that the repeated measures factor was added to the linear model of the experiment despite of its design as a sub-plot factor (Damon and Harvey 1987). We are interested in the answers to the two main questions: 1. How does the treatment mean change over time, i.e. is there a time main effect? and 2. How do treatment differences change over time, i.e. is there a treatment × time interaction? The choice of a method of the analysis is often dependent on covariance matrix structure. The sphericity of covariance matrix i.e. condition assuming the equality of variances of differences between time
points is necessary and sufficient for the results of univariate “split plot in time” analysis to be correct. In order to check if such an equality is true, one of the sphericity tests is usually recommended, for instance Mauchly’s, Harris’ or Grieve’s test (Howell 2010). -When the covariance matrix holds the sphericity condition, i.e. when the variances of differences between measurements are equal (Field 1998), the univariate analysis of variance for the split-plot model in time or in space can be applied, where the repeated measure factor is treated as the factor of the subject units (Linell Nemec 1996). -When this condition is not fulfilled this method needs corrections to the test function \( F \) (Wallenstein et al. 1980; Bathke 2009), since the \( F \) test becomes too liberal. Greenhouse and Geisser (G-G) as well as Huynh and Feldt (H-F) proposed corrections for degrees of freedom of \( F \) test for both the treatment and error (Warner, 2008). However, some statisticians have questioned the need for conducting any preliminary test of sphericity especially the most popular in statistical packages – Mauchly’s test. They argue that sphericity is almost always violated to some degree and thus researchers should universally correct for this violation by adjusting df with the G-G or the H-F estimates. Additionally, the Mauchly test is not robust to non-normality: small departures from multivariate normality in the population distribution can lead to artificially low or high Type I error. Furthermore, the Mauchly test is dependent on the sample size: for large samples, small violations of sphericity often produce significant Mauchly test results while for small samples, the power of this test is too low to detect large violations of sphericity (Cornell et. al 1992).

Multivariate analysis (MANOVA)

In the multivariate approach, successive response measurements made over time or space are considered as correlated dependent variables. Observations for each level of within-subject factor (time/space) is presumed to be a different dependent variable. MANOVA assumes there is an unstructured covariance matrix for dependent variables. The analysis of variance is based on the data transformed by orthogonal contrasts. This method has its limitations due to the assumptions for multivariate data (Krzyśko 2000; Morrison 2004) as well as some inconvenience such as the necessity of removing the whole vector of observations in case of the data with missing observations.-The difference between the number of samples (subjects) and the number of between-subjects treatment levels (groups) must be greater than the number of dependent variables. Other assumptions for MANOVA are independence of observations, multivariate normal distribution of the data and equality of covariance matrices. That way the structure of covariance matrix is not taken into account and the ordinary \( F \) test is used for verifying the null hypothesis for treatments impact on the mean value of a studied feature, while to test the significance of repeated measures the test statistic based on Wilks \( \Lambda \) (or other statistics – Morrison, 2004) is applied. In the matrices’ notation the general linear model of the experiment with measurements made in time or space is the same as the model without repeated measurements and might be described as follows: \( \mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{E} \), where \( \mathbf{Y} \) is the matrix of observations, \( \mathbf{X} \) is design matrix, \( \mathbf{B} \) the model parameters and \( \mathbf{E} \) errors matrix. The difference between these two models is in the matrix \( \mathbf{B} \), where there are, besides treatment effects, the effects of time points or space levels. Then, in order to check if experimental factors influence significantly on the difference between mean values of studied feature, the null hypothesis \( \mathbf{H}_0: \mathbf{LBM} = 0 \) should be verified against the alternative one \( \mathbf{H}_1: \mathbf{LBM} \neq 0 \), where contrast matrix \( \mathbf{L} \) describes the linear combination of the experimental factor parameters and \( \mathbf{M} \) describes the linear combination of the parameters connected with repeat measures. Therefore, the analysis can be conducted to include the following stages: 1) transformation of original data replacing the original observation matrix \( \mathbf{Y} \) by \( \mathbf{YM} \), 2) the application of between-subjects (univariate) analysis to the first column of \( \mathbf{YM} \) matrix in order to estimate the impact of treatments, 3) the application of within-subjects (multivariate) analysis to the rest of the columns of \( \mathbf{YM} \) in order to estimate the influence of repeated measurements factor and its interaction with experimental factors, 4) when the significant changes of a studied feature under impact of repeated measures are obtained the analysis of time trends is highly recommended.

Univariate analysis of variance for mixed model

On the contrary to the multivariate method, the structure of covariance matrix is a basis for the procedure for the mixed model. The whole process could be divided into two stages: 1) the modelling of covariance matrix to receive a matrix with the simplest structure and 2) the analysis of variance using generalized test function \( F \) (Littel et al. 1998). In order to fit covariance matrix structure into experimental data, the methods based on likelihood function are applied. There are two methods described by Harville (1977): maximum likelihood (ML) method or the more preferable one - restricted maximum likelihood method, which provide estimates for variance components maximizing the likelihood function. The information criterion of Akaike
(AIC) or Schwarz (SBC) is the measure of the goodness of fit of an estimated statistical model. The aim is to find the minimum of a function, which can be written as \(-2\ln(L) + k\phi(n)\), where \(L\) is a maximum of the likelihood function, \(k\)- number of estimated parameters and \(n\) is the number of observations. For Akaike criterion \(\phi(n)=2\) and for Shwarz’s one \(\phi(n)=\ln(n)\) (Gurka, 2006). The obtained estimators of variance components are used to model parameters estimation as well as to calculate the value of the test function for the null hypothesis. The standard mixed model can be described by the following linear function:

\[
y = Xb + Zu + e
\]  

where \(b\) is the vector of fixed parameters, \(u\) – vector of random parameters , \(e\) – vector of errors, \(X\) and \(Z\) – incidence matrices.

The expected value of the observation vector is equal to \(Xb\), and its covariance matrix could be described by unknown matrices \(G\) and \(R\):

\[
\Sigma = \text{Var}(y) = \text{Var}(Zu+e) = ZGZ' + R
\]  

Assumptions: \(u \sim N_p(0, G)\), i.e., multivariate normal with mean vector 0 and covariance matrix \(G\), \(e \sim N_p(0, R)\), i.e., multivariate normal with mean vector 0 and covariance matrix \(R\) (repeated measures structure), \(u, e\) are uncorrelated.

Parameters estimators are obtained on the basis of a general least squares method, in which the normal equations take the following form (Kariya and Kurata, 2004):

\[
\begin{bmatrix}
X'\hat{R}^{-1}X & X'\hat{R}^{-1}Z \\
Z'\hat{R}^{-1}X & Z'\hat{R}^{-1}Z + \hat{G}^{-1}
\end{bmatrix}
\begin{bmatrix}
b \\
u
\end{bmatrix}
= 
\begin{bmatrix}
X'\hat{R}^{-1}y \\
Z'\hat{R}^{-1}y
\end{bmatrix}
\]  

(3)

In practice, the unknown matrices \(R\) and \(G\) are replaced with their estimators obtained by the methods mentioned above and on the basis of (3) the following estimators of the mode parameters are received:

\[
\hat{b} = \left(X\hat{\Sigma}^{-1}X\right)^{-1}X\hat{\Sigma}^{-1}y \quad \text{and} \quad \hat{u} = \hat{G}Z\hat{\Sigma}^{-1}(y - X\hat{b})
\]  

(4)

The hypothesis \(H_0: Lb = 0\), where \(L\) is a contrast matrix taking out the appropriate comparisons from vector of fixed parameters \(b\), is verified on the basis of generalized test function \(F\) (Verbeke and Molenberghs, 2000):

\[
F = \frac{(L\hat{b})' (L(X\hat{\Sigma}^{-1}X)^{-1}L') (L\hat{b})}{\text{rank}(L)}
\]  

having an approximate F-Snedecor distribution with \(\text{rank}(L)\) and \(v\) degrees of freedom. When the covariance matrix does not hold the sphericity assumption, the value \(v\) is an approximate number of degrees of freedom obtained for instance by the application of Satterthwaite procedure (Sarhai and Ojeda 2005).

**Procedure for the analysis method choice**

The schema of repeated measurements data analysis is presented below (Fig 1). A number of assumptions have to be checked to use the chosen method of analysis. If the data do not meet these assumptions a researcher is obliged to apply a different method of analysis. At every step of the analysis he should have information of "what if?".

The following procedure is designed to choose the simplest method for the data that meet specific criteria.
Fig. 1 The schema of procedure of analysis choice for repeated measurements.

First of all the basic assumptions of normal distribution and homogeneity of variance have to be checked to decide if parametric methods are possible to use. The mixed model is the most flexible one and is possible to apply it, even if the data does not meet these assumptions. However, as presented in Fig. 1, the non-parametric methods or the data transformation are better choice in this case. The MANOVA approach is desirable when the experimental model is complicated and the researcher is not interested in the discovery of the repeated measurements correlations. Otherwise the mixed model is the better choice as the one that shows the structure of covariance matrix. When the model of the experiment is quite simple the ANOVA with Greenhouse and Geisser or Huynh and Feldt corrections for degrees of freedom of F test is recommended. The meeting of sphericity assumption by data allowed us to perform the simplest ANOVA.

Discussion

This review of the three most popular parametric methods does not cover all of the ones used in these types of issues. A broader presentation of the problem can be found in Keselman et al. (2001), where additional methods such as empirical Bayes approach, or the Welch-James adjusted F test are presented. In that paper one can also find the information on the robustness of the particular tests to failure of analysis of variance assumptions.

When the observations do not follow a Gaussian distribution a nonparametric approach is desirable (Bruner et al. 2002, Munzel et Tamhane 2002). The nonparametric methods for this problem use multiple sign tests and rank tests (Akritas 1991, Bruner et al. 1999). It should be mentioned, however, that in case of normal-distributed data the power of nonparametric tests is lower than the power of parametric ones.

In the present research on the repeated measures studies a nonparametric approach papers can be found (Brunner at al. 2002, Bathke et al. 2008, Bathke and Harrar 2008, Harrar and Bathke 2008), where the hypotheses are not formulated in terms of expectations of treatment effects, but rather in terms of their distribution functions. As applied to rank transformed data these methods do not require distributional assumptions, and can be applied to a variety of data types for instance continuous, discrete or ordinal. They are also robust with respect to outliers and for small sample sizes. The nonparametric alternative to “split plot in time” (the first example) design is F2-LD-F1 design proposed by Brunner et al. (2002) while the
multivariate approach in repeated measures issue was described by Bathke and Harrar, who considered rank test statistics based on separate rankings for the variables determined by the time points for both - balanced (Bathke and Harrar 2008) and unbalanced (Harrar and Bathke 2008) designs.

Conclusions

1. Selection of data analysis method with repeated measurements requires checking of the many assumptions as well as determining the range of received information. The developed procedure of method selecting hepls researchers to make a correct choice on the basis of information from the experiment.

2. The application of ANOVA with corrections for degrees of freedom of $F$ test (despite sphericity test results) is recommended as the easiest one in case when assumptions of normal distribution of the data as well as homogeneity of variance are held.

3. Since there is a possibility of omitting the problem with covariance matrix structure, the multivariate method could be applied when the model of the experiment is complicated (for instance split-plot design). However, this kind of analysis cannot be recommended when the number of missing observations is high, the data distribution is not normal or the covariance matrices are not homogeneous.

4. Mixed models method is the most complicated and time consuming one. On the other hand it is also the most robust to covariance heterogeneity and non-normality and provides additional information about correlation matrix structure, therefore it can be used when the data do not hold the ANOVA assumptions or the relationships between measurements are the subject of interest.

5. The nonparametric tests (examples not presented here) are a good alternative to the parametric ones in the case of non-normal distribution and small samples case.

Abstract

In the integrated logistics and marketing management the company increasingly adopting the results of modern industrial experiments. The paper presents the principle of selecting one of these types statistical analysis method. The aim of the study was to determine the conditions for application of a few selected methods for analyzing of repeated measurements data and to develop procedures for dealing with decision-making process of choosing one of them. Analysis of this type of data is often carried out in an irregular manner, in particular correlations between measurements are not always taken into account. Three basic parametric methods for the analysis of such data were considered and discussed. The decisional procedures for the methods of analysis with repeated measurements are presented graphically in the form of a diagram. The developed procedures for handling enriched with practical tips for choosing the methods of data analysis with repeated measurements, which were formulated in the form of conclusions.

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