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Maximisation of Sales Volume with Respect to the Company’s Value

Introduction

The literature of the subject presents a number of concepts of the company’s value. Leaving the definitional details aside, the company’s value can be determined by a mathematical model. The construction of this model should take into account the key factors that create the company’s value.

A company is a specific form of investment. Investing their capital resources, the owners expect to obtain benefits in the form of the increased value of the invested capital. Applying this perspective we can perceive a company as a subject of valuation and indicate the determinants of its value. Borowiecki (1998) claims that: the level of the company’s market value can be differentiated being dependent to a large extent on the value of its assets referred to as the company’s real capital. This means that the company’s value based on the value of its assets is not equivalent to a simple sum of particular components of the company’s assets less the value of the incurred liabilities. In the market economy conditions the company can be viewed from the perspective of the scale and volume of the sales of products which it manufactures. Traditionally the aim of the company’s functioning has been to maximise profits. The aim of the modern company, however, in terms of strategy, is becoming the growth of the company’s market value.

The aim of the present article is to present the model approach towards the creation of the company’s value in terms of strategy. The reasoning is focused on making the relationships between the company’s value and maximisation of the volume of sales more comprehensible.

The merits of the article are based on the literature study of the subject, the author’s empirical research and discussions on the subject in the business environment.

THE COMPANY’S VALUE AS A STRATEGIC AIM WITH RESPECT TO THE MAXIMISATION OF THE COMPANY’S VOLUME OF SALES

The key role in the effective increase of the volume of sales is played by the logistics system of distribution. The presence of the manufactured goods on the market and their effective sales are a key function of the company performed by the company’s system of distribution. Companies create their distribution systems in many diverse ways. They should be adjusted, however, to the specific conditions of the company’s functioning and the assumed goals.

The increase of the company’s volume of sales, reflects, among others, the company’s market success. It should be remembered, however, that the maximisation of the company’s sales is not always equivalent to the maximisation of the company’s profits. Therefore, sales intensification should be conducted under the condition that the profits made are not lower than the minimum level determined by the owner. The presented procedure of economic modelling is essentially about searching for such a volume of sales that brings the owner (entrepreneur) the assumed profit. It is worth noting that from the point of view of creating the company’s value it does not have to be the maximum profit. The sufficient level of profit is the one that allows to maintain the company’s status quo in terms of its value and is satisfying for the owners. It should be made sure that the profit level does not fall below a certain fixed minimum, e.g. Z0, which is set below the maximum profit level related to the conditions MR = MC.

The starting point in the presented argument is the condition that the marginal revenue equals the marginal cost (MR = MC). Under this assumption the initial sales maximisation model takes the following form:

\[ S = S(P) \]  

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3. Article reviewed.
4. R. Borowiecki (and others), Metody i procedury wyceny przedsiębiorstw i ich majątku (Valuation methods and procedures of the company and its assets), Wydawnictwo Profesjonalnej Szkoły Biznesu, Kraków, 1998.
5. S. Mynarski, Badania rynkowe w warunkach konkurencji (Market research under conditions of market competition), Oficyna Wydawnicza „Fogra”, Kraków 1995.
under the condition that: \( Z = S(P) - K(P) \geq Z_0 \)

where:
- \( S \) – sales volume,
- \( S(P) \) – sales function described by the volume of production,
- \( K(P) \) – cost function described by the volume of production,
- \( Z \) – maximised profit,
- \( Z_0 \) – assumed minimum profit,
- \( P \) – volume of production.

So the model under consideration takes the following formula:

\[ S(P) \rightarrow \text{maximum} \]
\[ Z = K(P) - S(P) \leq -Z_0 \]  \hspace{1cm} (2)

(the condition resulting from the transformation of the side condition)

\[ P \geq 0 \] \hspace{1cm} (4)

The proposed concept of the optimisation model makes it necessary to use Kuhn-Tucker conditions\(^8\). When we use Lagrange function the profit model becomes the following equation:

\[ Z = S(P) + \lambda(-Z_0 - K(P) + S(P)) \] \hspace{1cm} (1a)

while using Kuhn-Tucker conditions leads to the marginal conditions:

\[ \frac{\partial Z}{\partial P} = S'(P) - \lambda K'(P) + \lambda S'(P) \leq 0 \] \hspace{1cm} (2a)
\[ \frac{\partial Z}{\partial \lambda} = -Z_0 - K(P) + S(P) \geq 0 \] \hspace{1cm} (3a)

where:
- \( K'(P) \) – marginal cost
- \( S'(P) \) – marginal sales

and also non-negativity and complementarity constraints.

The case that is important for further reasoning is when \( P = 0 \). Then \( S(0) = 0 \) and \( K(0) > 0 \), for the ‘zero’ level of production we obtain:

\[ \frac{\partial Z}{\partial \lambda} = -Z_0 - K(0) < 0 \] \hspace{1cm} (5)

which contradicts the marginal condition (3a) for \( P > 0 \). Therefore it is necessary to assume that \( P > 0 \), as the ‘zero’ level of production is outside the range of acceptable solutions. The fact that the volume of production is a positive number implies that pursuant to the condition of complementarity, the inequality (3a) should be transformed into the equation form. Then, solving the equation, we will obtain the volume of production that maximises the sales.

\[ S'(P) = \frac{\lambda}{1+\lambda} \cdot K'(P) \] \hspace{1cm} (6)

Equation (6) expresses the rule of control over the production volume. There are, however, some constraints to this rule. For example, we cannot assume that the function used in constrain (2) is quasi-convex. It is significant as this doesn’t allow us to automatically use Arrow-Enthoven sufficiency theorem\(^9\) about the condition concerning the multivariate value of \( \lambda \). Assuming that \( \lambda = 0 \) we obtain \( S'(P) = 0 \), which means that at that point the sales curve reaches its peak point, i.e. sales will be maximised\(^10\). There is no certainty, however, that having the maximum sales the company will make the maximum profit.

Assuming \( \lambda > 0 \) we will obtain a constraint of sales maximisation for the optimum level of production that maximises profit. Then formula (6) assumes the following form:

\[ S'(P) \geq \frac{\lambda}{1+\lambda} \cdot K'(P) \] \hspace{1cm} (7)

and expresses the minimum profit required for a given volume of production.

Discussing the model maximising sales we need to take into account the product lifecycle, that is to find the point on the product lifecycle curve at which a new product should be created. In order to do that we should carry out analysis applying a numerical model of the following kind:

\[ S(P) = 10P - P^2 + 5 \rightarrow \text{sales function (concave function)} \]

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\(^8\) The assumption that function (1) is differentiable and concave implies that the cost function is differentiable and convex, which means that function (2) is differentiable and convex and that is why we can use Kuhn-Tucker theorem.

\(^9\) A. C. Chiang, Podstawy ekonomii matematycznej (Basics of mathematical economy), op. cit., p. 742.

\(^10\) A. C. Chiang, Podstawy ekonomii matematycznej (Basics of mathematical economy), op. cit., p. 748.
According to Kuhn-Tucker conditions\(^{11}\) the initial model ((1a) – (3a)) will take the form of the Lagrangian function:

\[
Z = 10P - P^2 + 5 + \lambda[-Z_0 - (P^2 + 5P + 3) + (10P - P^2 + 5)]
\]

whereas Kuhn-Tucker conditions will constitute a set of marginal values of the profit function.

\[
\frac{\delta Z}{\delta \lambda} = 10 - 2P - \lambda(2P + 5) + \lambda(10 - 2P) \leq 0.
\]

and then after simplification it will be:

\[
\frac{\delta Z}{\delta \lambda} = -Z_0 - (P^2 + 5P + 3) + (10P - P^2 + 5) \geq 0
\]

Solving the set of equations [4] - [5] and taking into account the principle of mutual complementarity and the condition \(p > 0\) we obtain an equation with two unknown quantities. Only \(\lambda\) is subject to variation changes. Assuming that \(\lambda > 0\) (according to the principle of complementarity \(\frac{\delta Z}{\delta \lambda} = 0\)), the solution of the set of equations [4] - [5] are two roots: \(P_1 = 2.68\) and \(P_2 = -0.2\).

In accordance with the set goal, i.e. maximisation of sales, we should assume that \((S'(P) = K'(P))\) \(P_1 = 2.68\). It should be noted that \(P_2\) does not satisfy equation [5]. Applying the optimisation rule results in obtaining a lower value of production \(P = 1.25\).

Drawing 1 below\(^{12}\) presents the optimisation results of sales maximisation under the condition of the minimum profit discussed in the article.

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\(^{11}\) W. I. Zangwill, Programowanie nieliniowe (Non-linear programming), WMT, Warszawa 1974, p. 45.

\(^{12}\) A. C. Chiang, Podstawy ekonomii matematycznej (Basics of mathematical economy), op. cit., p. 748.
Part (a) of Drawing 1 presents costs and sales curves and the volume of sales \( P^*(\text{opt}) \) maximising the \( P^*(Z_{\text{max}}) \). Part (b) of Drawing 1 presents the interval of permissible production values determined by the difference between the marginal costs of production and the marginal sales exposing the optimal production \( P_4^* \) that gives the maximum volume of sales under the condition of the assumed minimum profit.

On the basis of the mathematical analysis of the function used in the constraint that is a mirror reflection of the profit curve in relation to the horizontal axis it can be concluded\(^{13}\) that every level of production in the open interval \((P_3, P_5)\) creates a positive profit, but only the values of \( P \) belonging to the closed interval \((P_2, P_4)\) yield the profit that is not lower than \( Z_0 \).

Constraint (2) should be met at the same time. Under this assumption the interval \((P_2, P_4)\) will finally be a set of permissible solutions. This means that it also comprises \( P_3 \), which is the volume of production that maximises the profit.

**CONCLUSIONS**

1. In modelling the strategy of the company’s growth and the strengthening of its position on the market by increasing the company’s value it is essential to take into account the key factors that create the company’s value.
2. While constructing the model of sales maximisation it is necessary to use fairly complex tools of the maximization calculus, the so-called Kuhn-Tucker conditions.
3. The strategy of the sales value maximization is a significant factor of the company’s growth and the creation of its market value.

**Abstract**

The article deals with the problems of the modelling approach towards the dependence of the growth of the company’s market value on maximisation of its volume of sales. The company’s production should be intensified under the condition that the company’s profit remains at least at the minimum level. The optimisation calculations carried out with the use of complex mathematical tools lead to the creation of the model of sales maximisation. Solving this model is equivalent to defining the volume of production that enables to create the company’s value as a strategic aim.

**Key words**: model of growth, strategic growth, sales maximisation, Kuhn-Tucker conditions

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\(^{13}\) A. C. Chiang, *Podstawy ekonomii matematycznej* (Basics of mathematical economy), op. cit., p. 748.